

# Chapter 6:

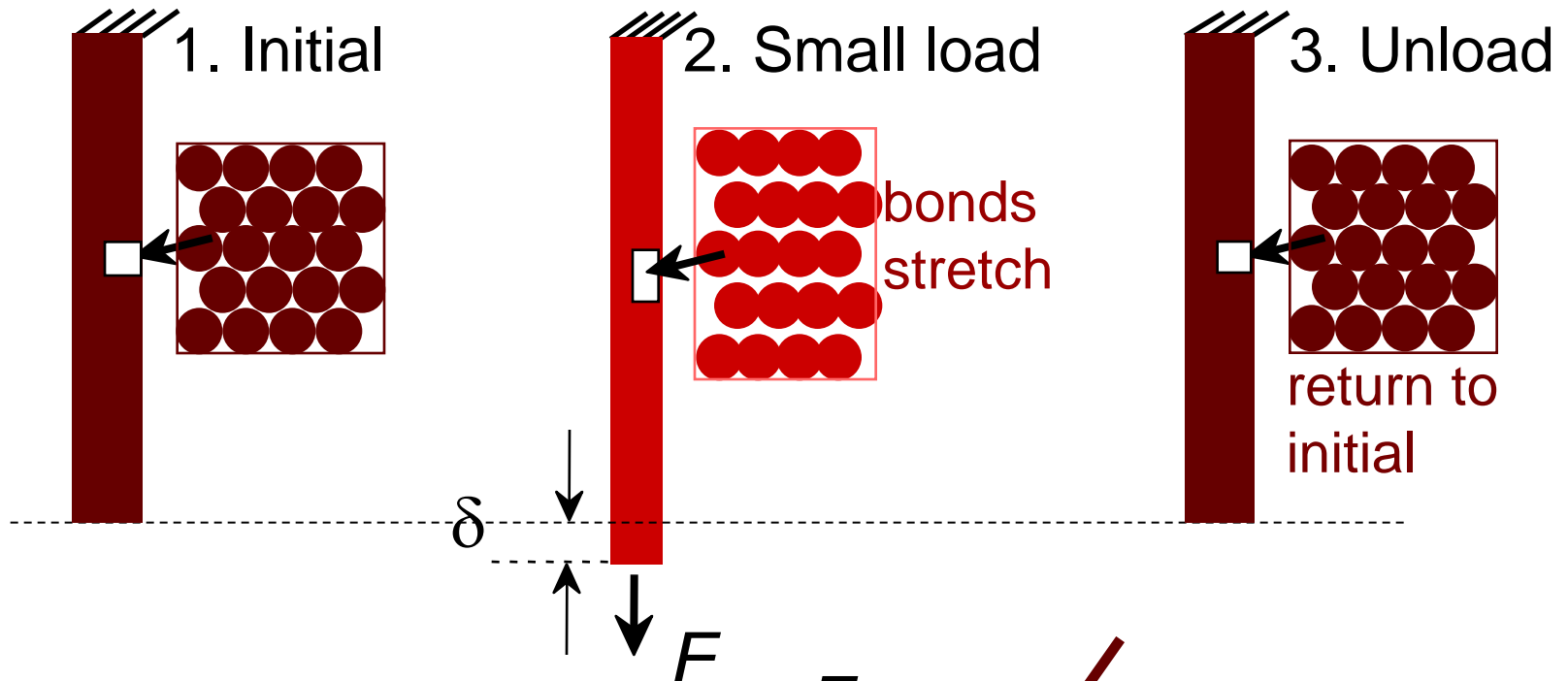
# Mechanical Properties

## ISSUES TO ADDRESS...

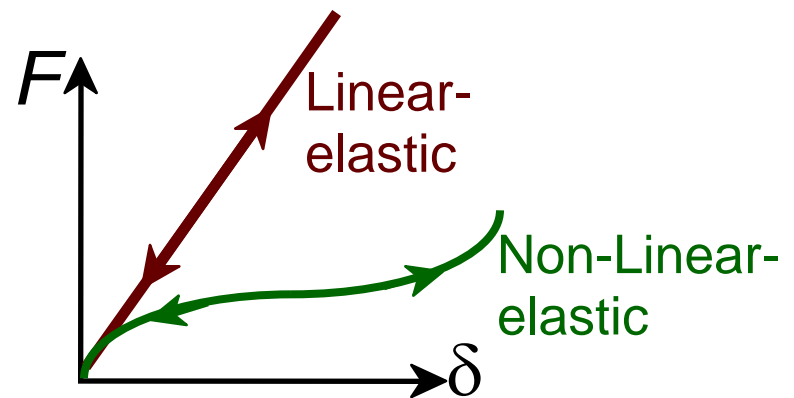
- **Stress** and **strain**: What are they and why are they used instead of load and deformation?
- **Elastic** behavior: When loads are small, how much deformation occurs? What materials deform least?
- **Plastic** behavior: At what point does permanent deformation occur? What materials are most resistant to permanent deformation?
- **Toughness** and **ductility**: What are they and how do we measure them?



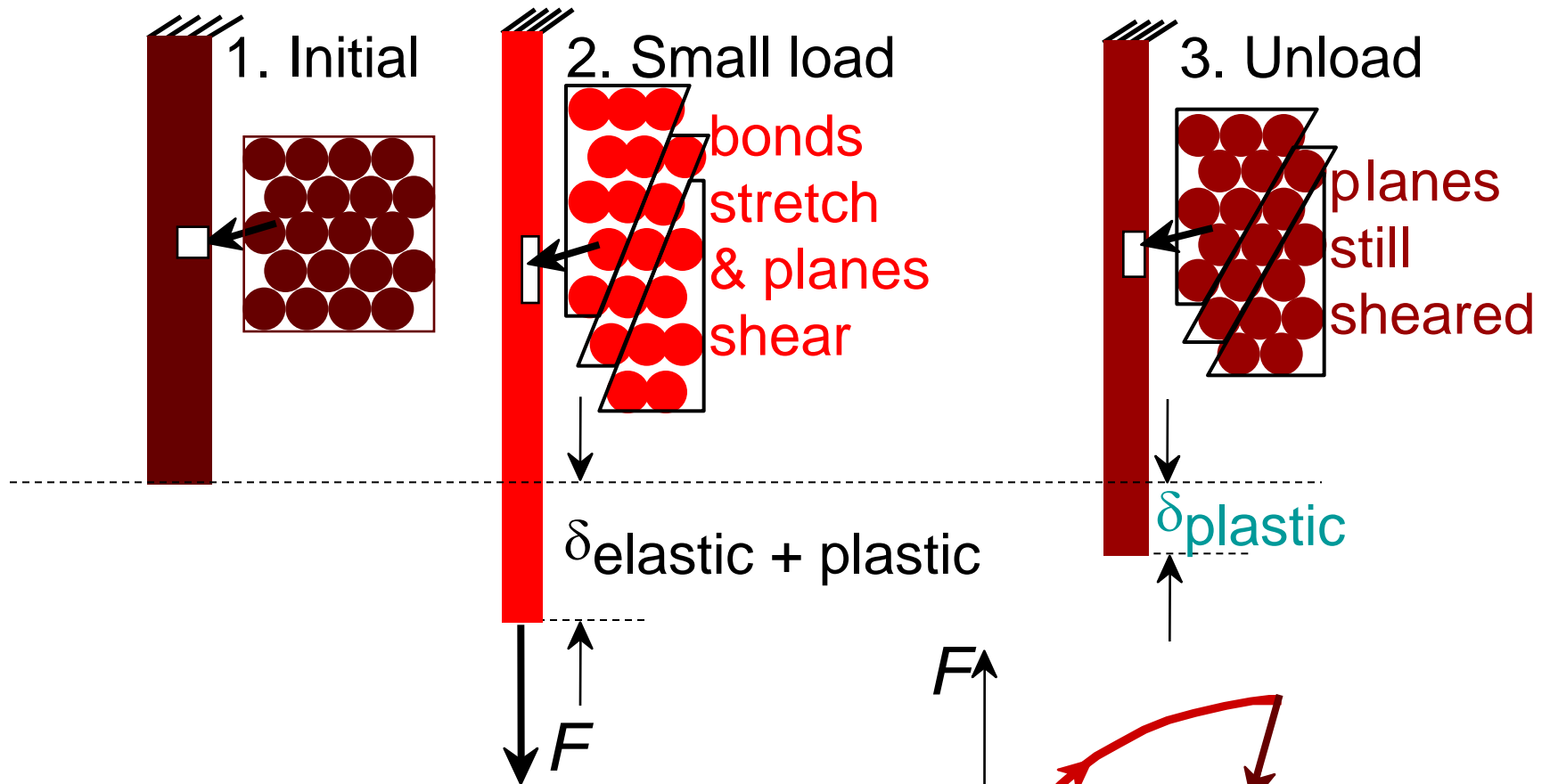
# Elastic Deformation



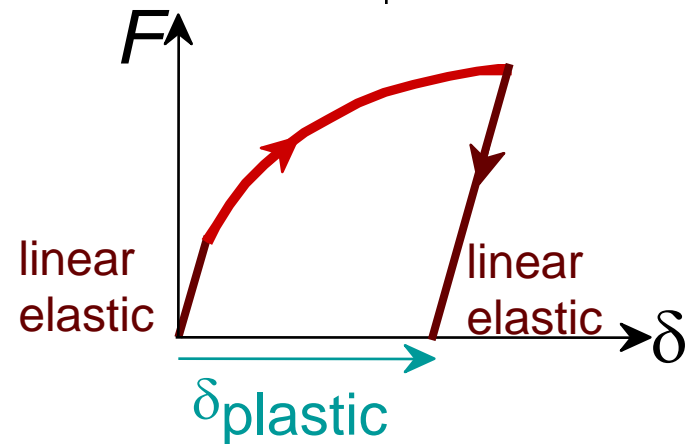
Elastic means **reversible**!



# Plastic Deformation (Metals)

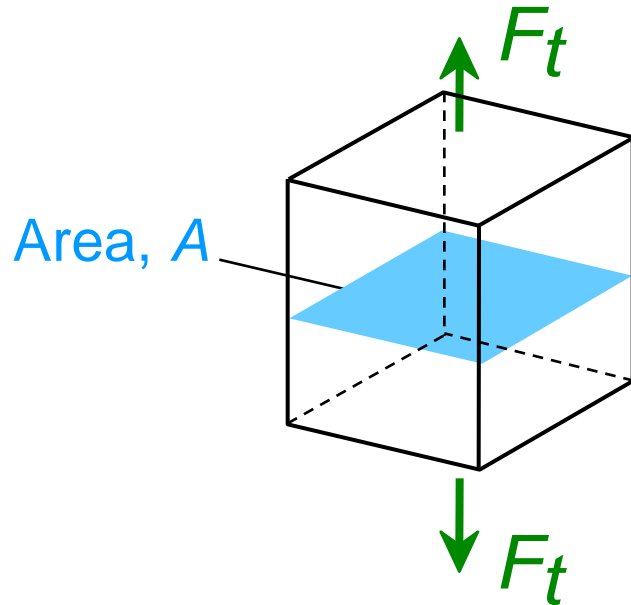


Plastic means **permanent!**



# Engineering Stress

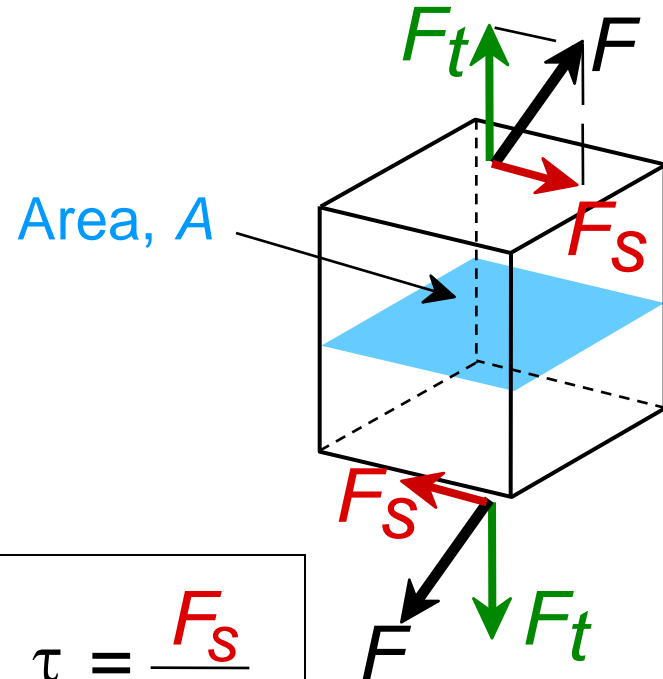
- Tensile stress,  $\sigma$ :



$$\sigma = \frac{F_t}{A_o} = \frac{\text{lb}_f}{\text{in}^2} \text{ or } \frac{\text{N}}{\text{m}^2}$$

original area  
before loading

- Shear stress,  $\tau$ :



$$\tau = \frac{F_s}{A_o}$$

$\therefore$  Stress has units:  
 $\text{N/m}^2$  or  $\text{lb}_f/\text{in}^2$



# Common States of Stress

- **Simple tension:** cable



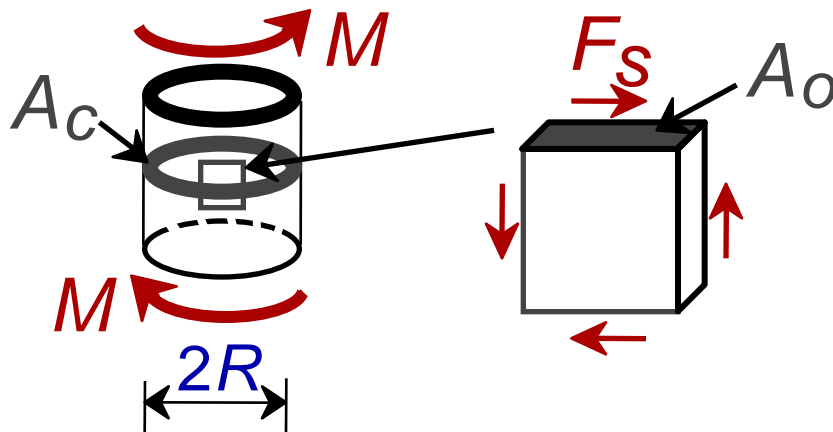
$A_0$  = cross sectional area (when unloaded)

$$\sigma = \frac{F}{A_0}$$

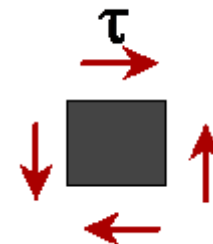


- **Torsion** (a form of shear): drive shaft

Ski lift (photo courtesy P.M. Anderson)



$$\tau = \frac{F_s}{A_0}$$

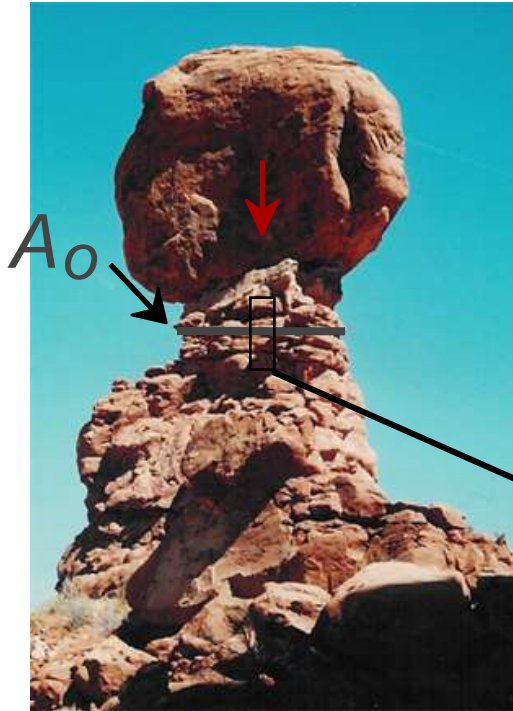


Note:  $\tau = M/A_c R$  here.



# OTHER COMMON STRESS STATES (1)

- **Simple** compression:



Balanced Rock, Arches  
National Park  
(photo courtesy P.M. Anderson)



Canyon Bridge, Los Alamos, NM  
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$



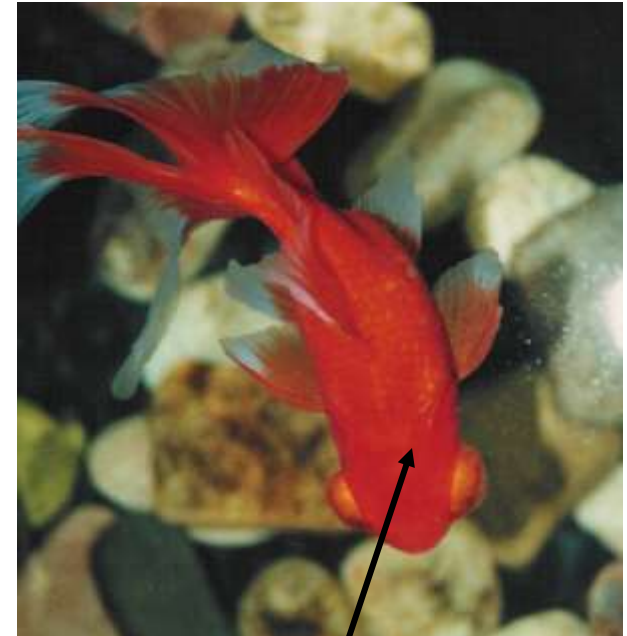
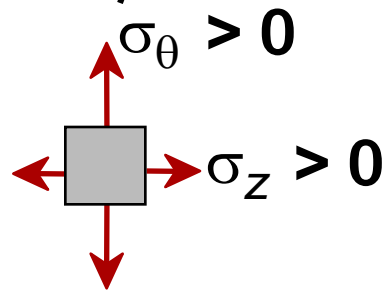
Note: compressive  
structure member  
( $\sigma < 0$  here).

# OTHER COMMON STRESS STATES (2)

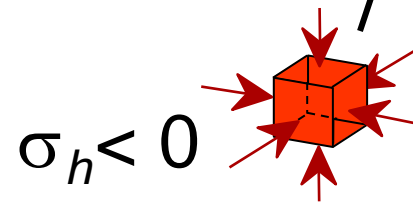
- **Bi-axial** tension:
- **Hydrostatic** compression:



Pressurized tank  
(photo courtesy  
P.M. Anderson)



Fish under water  
(photo courtesy  
P.M. Anderson)



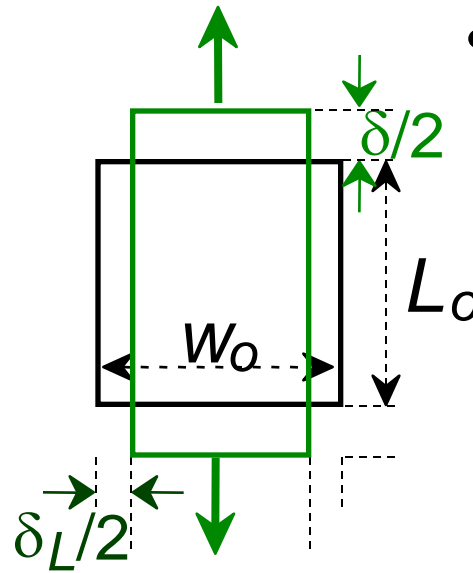
# Engineering Strain

- **Tensile strain:**

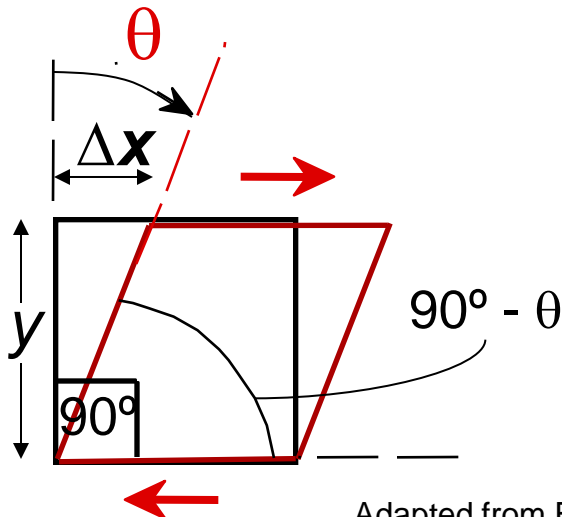
$$\varepsilon = \frac{\delta}{L_o}$$

- **Lateral strain:**

$$\varepsilon_L = \frac{-\delta_L}{W_o}$$



- **Shear strain:**



$$\gamma = \Delta x / y = \tan \theta$$

**Strain is always dimensionless.**

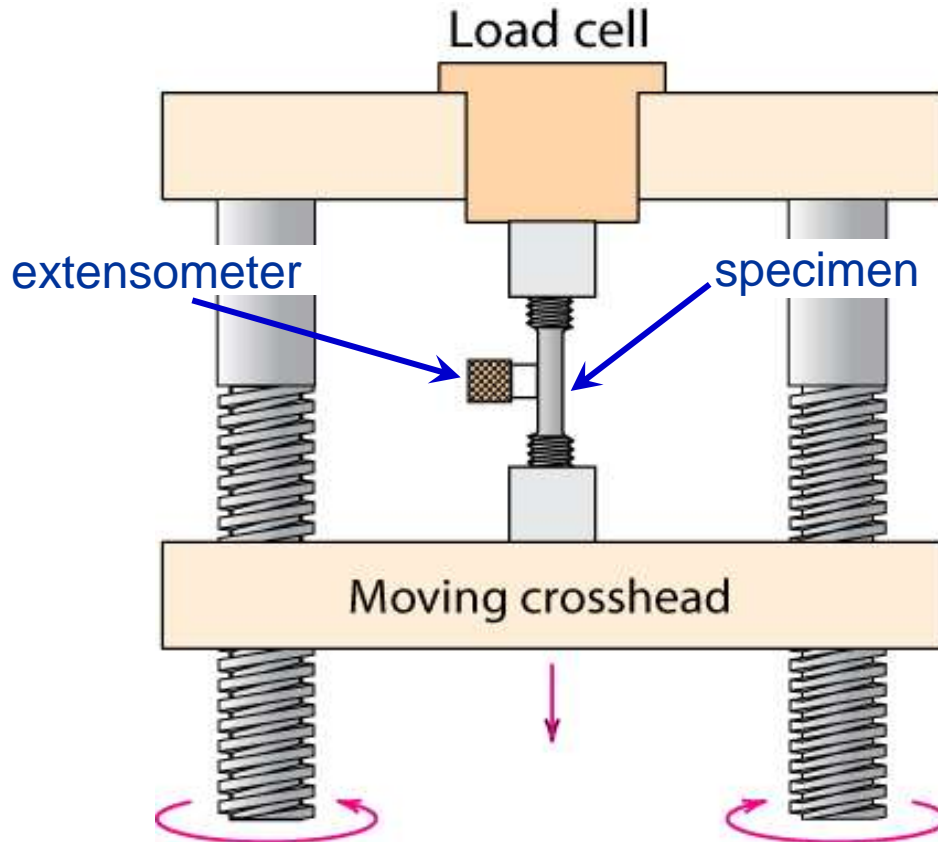
Adapted from Fig. 6.1 (a) and (c), Callister 7e.





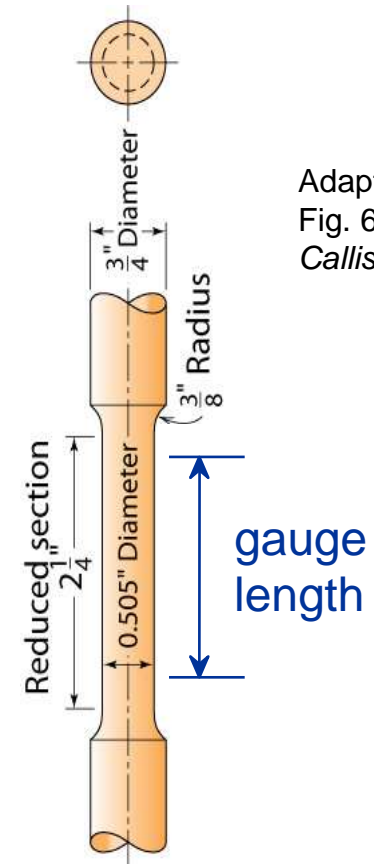
# Stress-Strain Testing

- Typical tensile test machine



Adapted from Fig. 6.3, *Callister 7e*. (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

- Typical tensile specimen



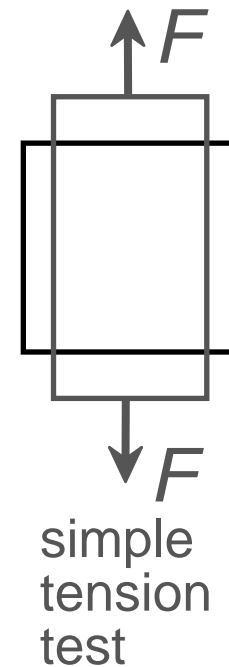
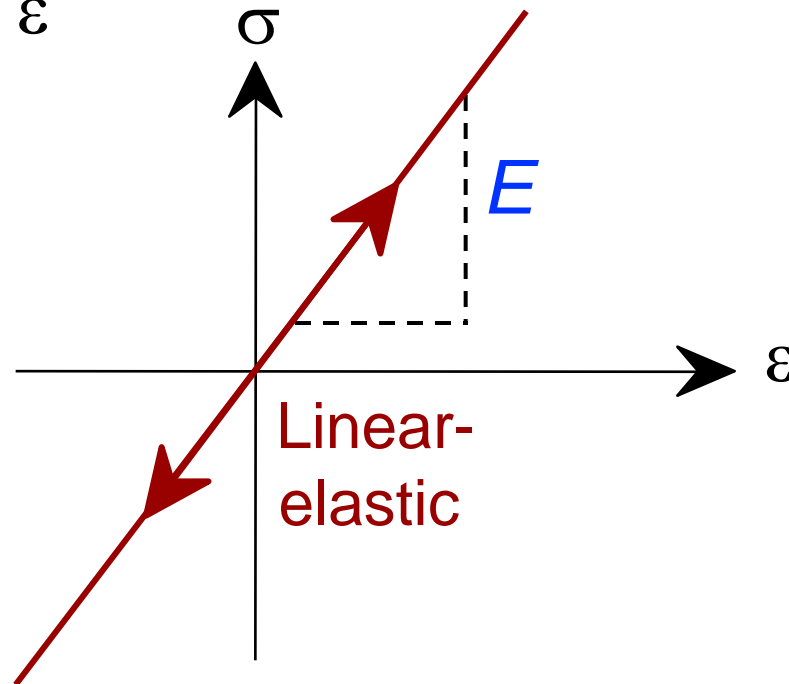
Adapted from  
Fig. 6.2,  
*Callister 7e*.



# Linear Elastic Properties

- **Modulus of Elasticity,  $E$ :**  
(also known as Young's modulus)
- **Hooke's Law:**

$$\sigma = E \varepsilon$$



# Poisson's ratio, $\nu$

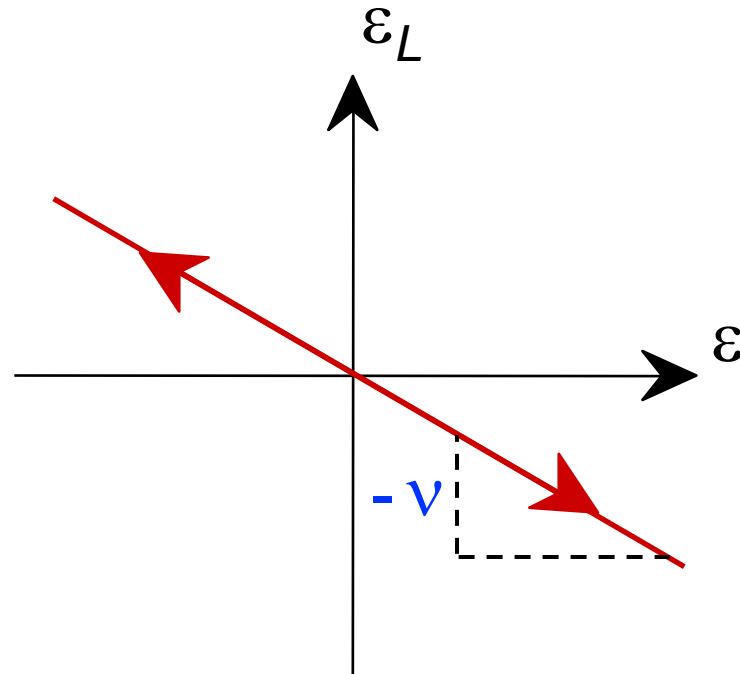
- Poisson's ratio,  $\nu$ :

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

metals:  $\nu \sim 0.33$

ceramics:  $\nu \sim 0.25$

polymers:  $\nu \sim 0.40$



Units:

$E$ : [GPa] or [psi]

$\nu$ : dimensionless

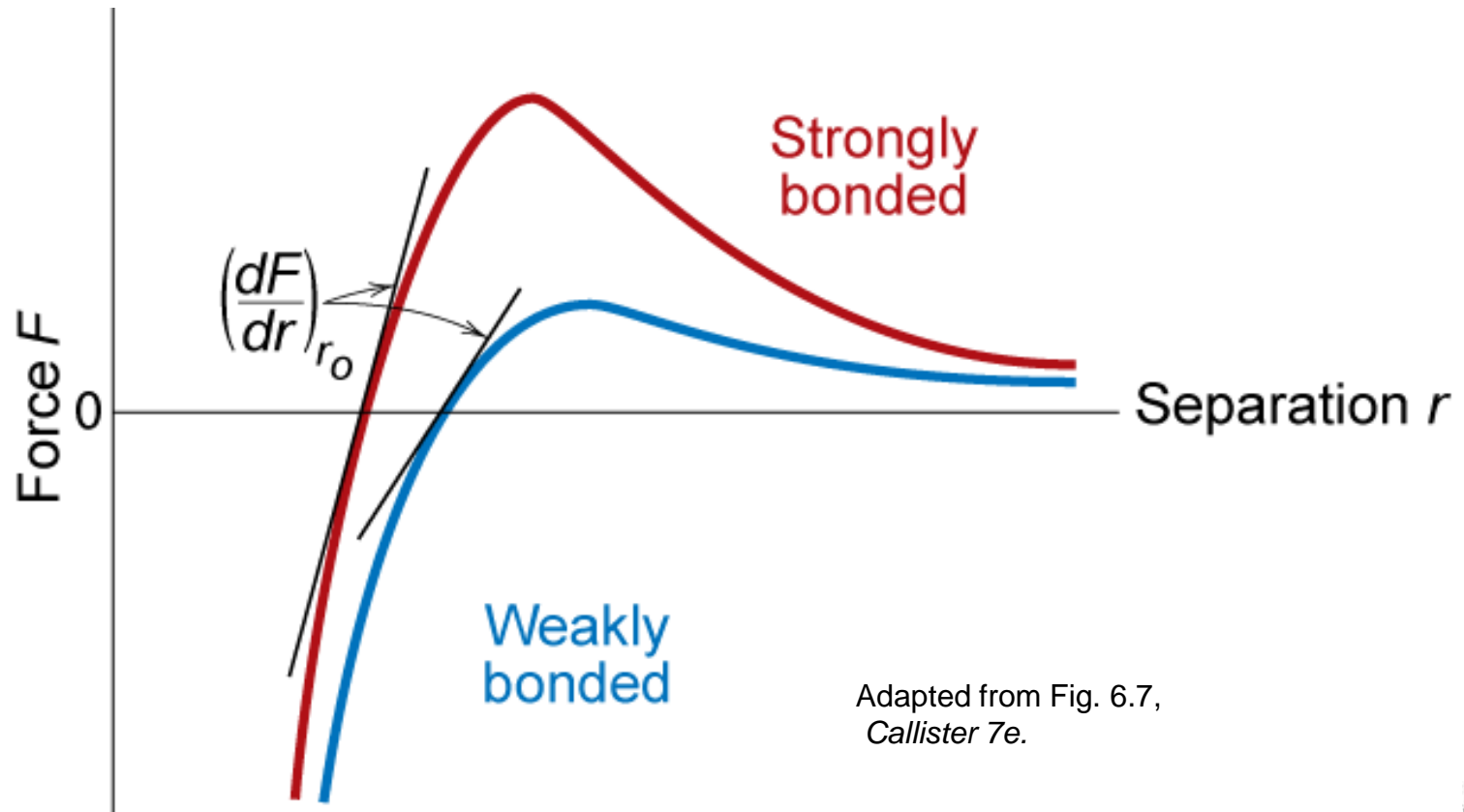
$-\nu > 0.50$  density increases

$-\nu < 0.50$  density decreases  
(voids form)



# Mechanical Properties

- Slope of stress strain plot (which is proportional to the elastic modulus) depends on bond strength of metal



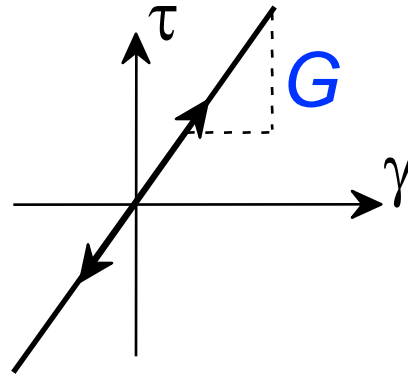
Adapted from Fig. 6.7,  
*Callister 7e.*



# Other Elastic Properties

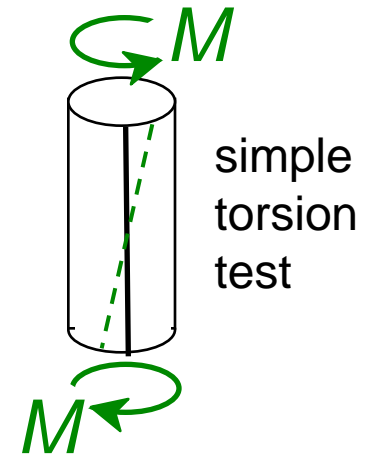
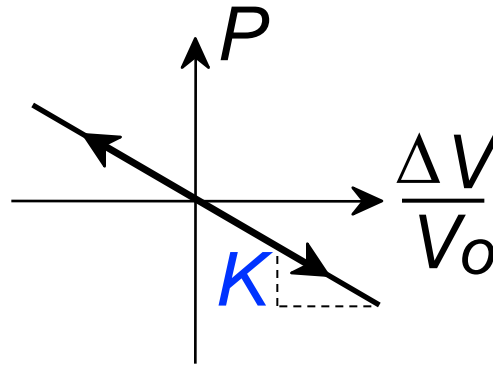
- Elastic Shear modulus,  $G$ :

$$\tau = G \gamma$$

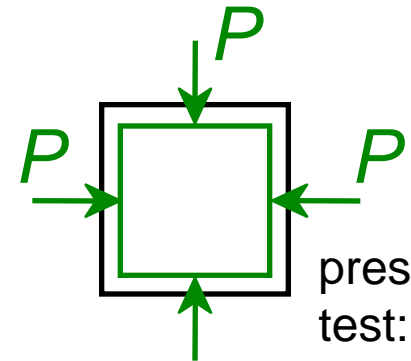


- Elastic Bulk modulus,  $K$ :

$$P = -K \frac{\Delta V}{V_0}$$



simple  
torsion  
test



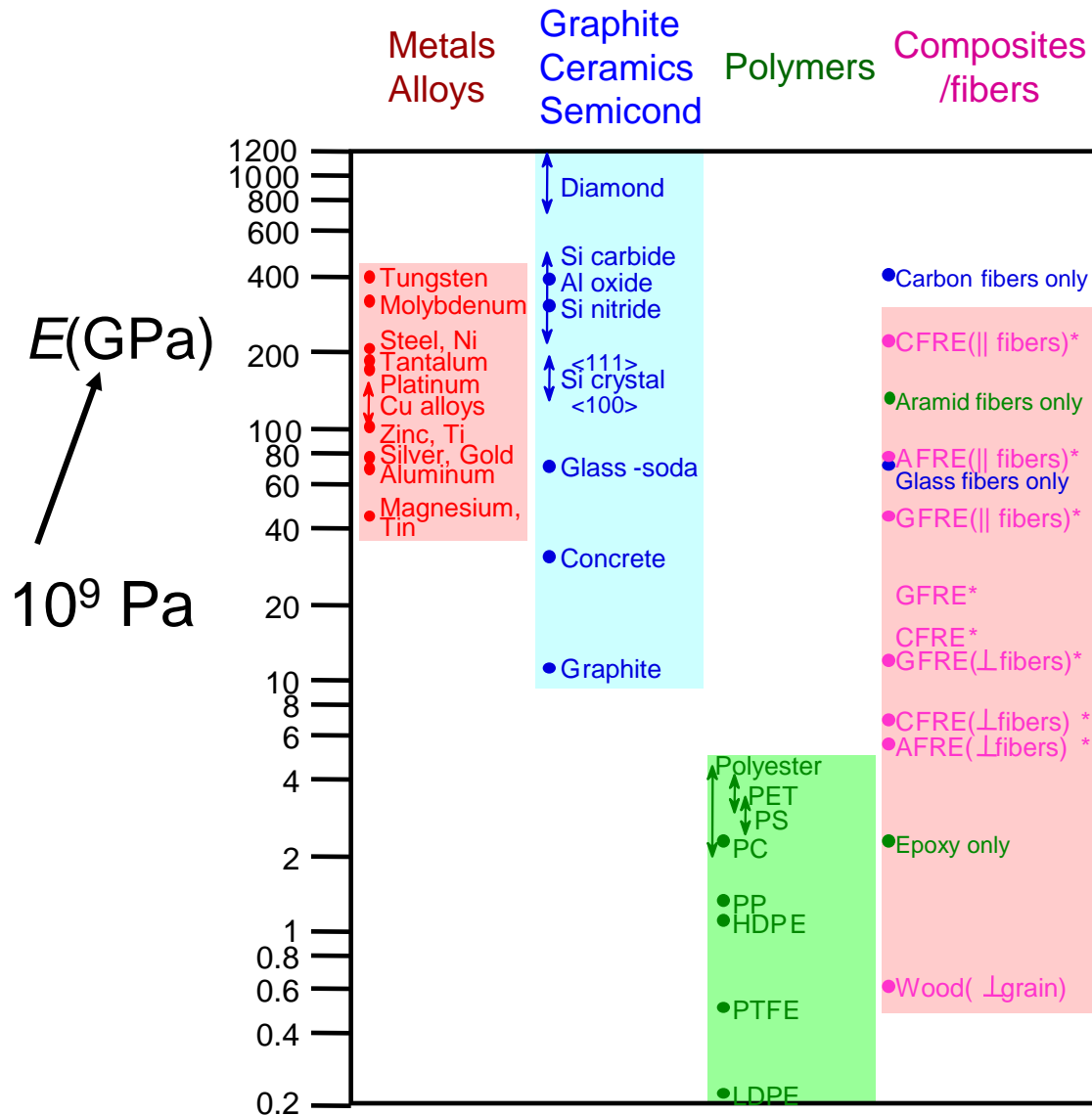
pressure  
test: Init.  
vol =  $V_0$ .  
Vol chg.  
=  $\Delta V$

- Special relations for isotropic materials:

$$G = \frac{E}{2(1 + \nu)}$$

$$K = \frac{E}{3(1 - 2\nu)}$$

# Young's Moduli: Comparison



Based on data in Table B2,  
*Callister 7e.*

Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

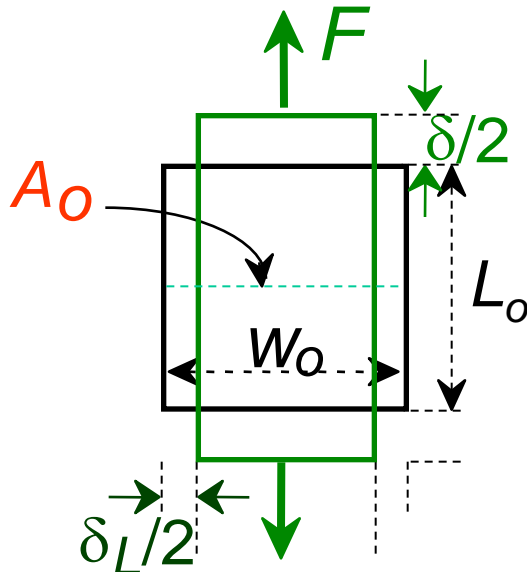


# Useful Linear Elastic Relationships

- Simple tension:

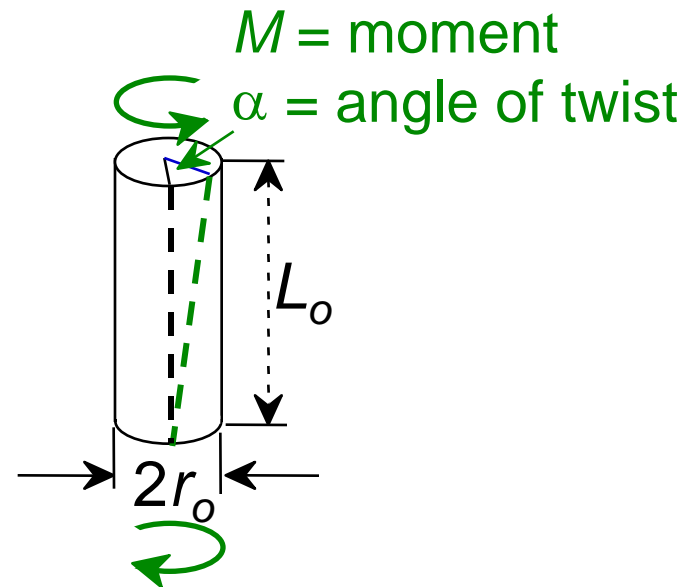
$$\delta = \frac{FL_o}{EA_o}$$

$$\delta_L = -\nu \frac{FW_o}{EA_o}$$



- Simple torsion:

$$\alpha = \frac{2ML_o}{\pi r_o^4 G}$$



$M$  = moment

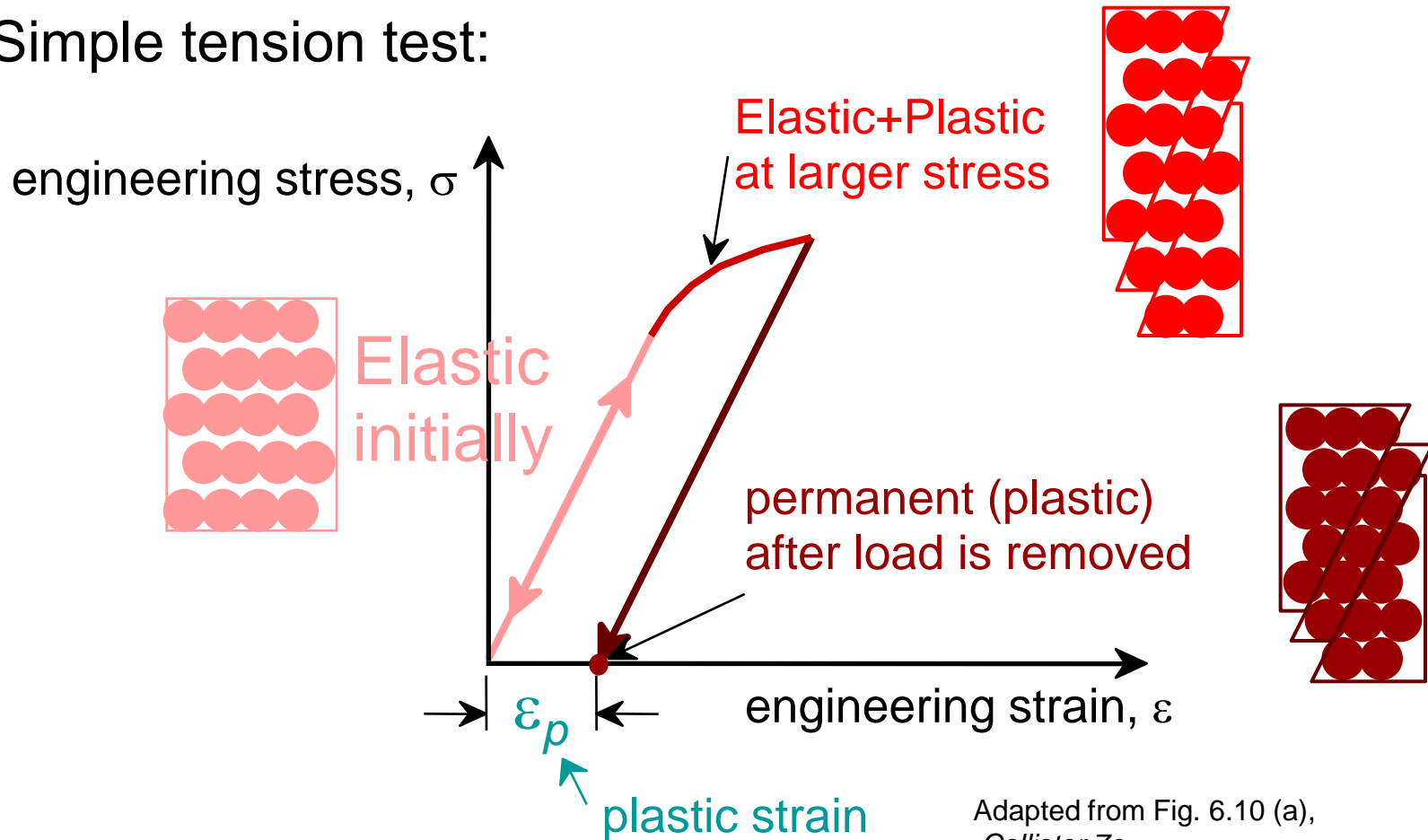
$\alpha$  = angle of twist

- Material, geometric, and loading parameters all contribute to deflection.
- Larger elastic moduli minimize elastic deflection.

# Plastic (Permanent) Deformation

(at lower temperatures, i.e.  $T < T_{melt}/3$ )

- Simple tension test:



Adapted from Fig. 6.10 (a),  
*Callister 7e*.

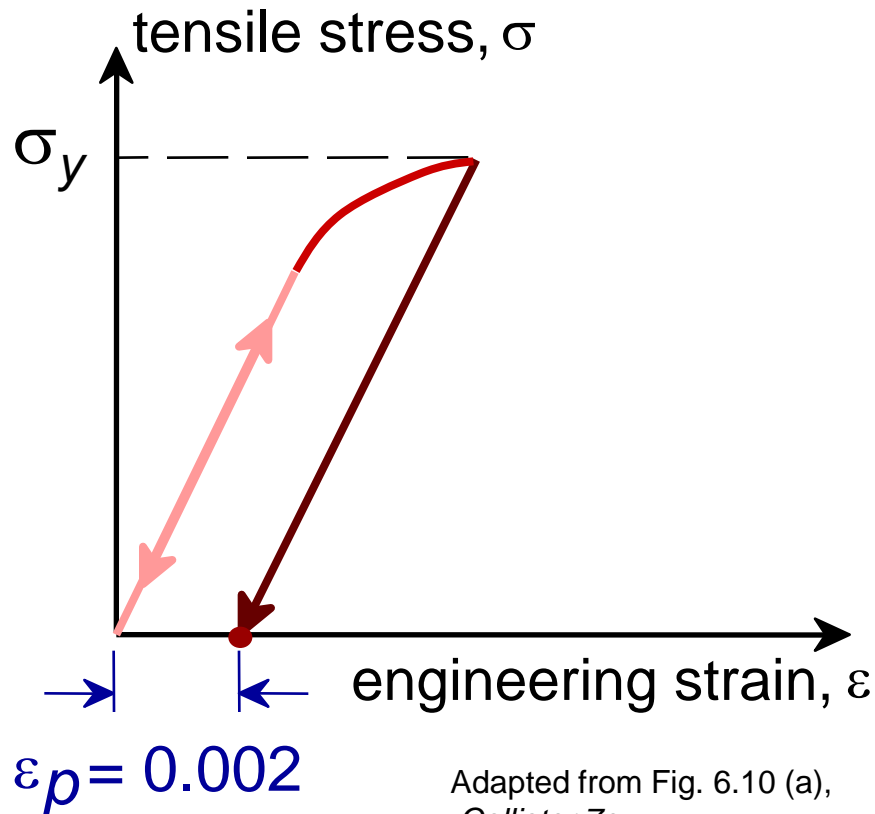




# Yield Strength, $\sigma_y$

- Stress at which *noticeable* plastic deformation has occurred.

when  $\varepsilon_p = 0.002$



$\sigma_y$  = yield strength

Note: for 2 inch sample

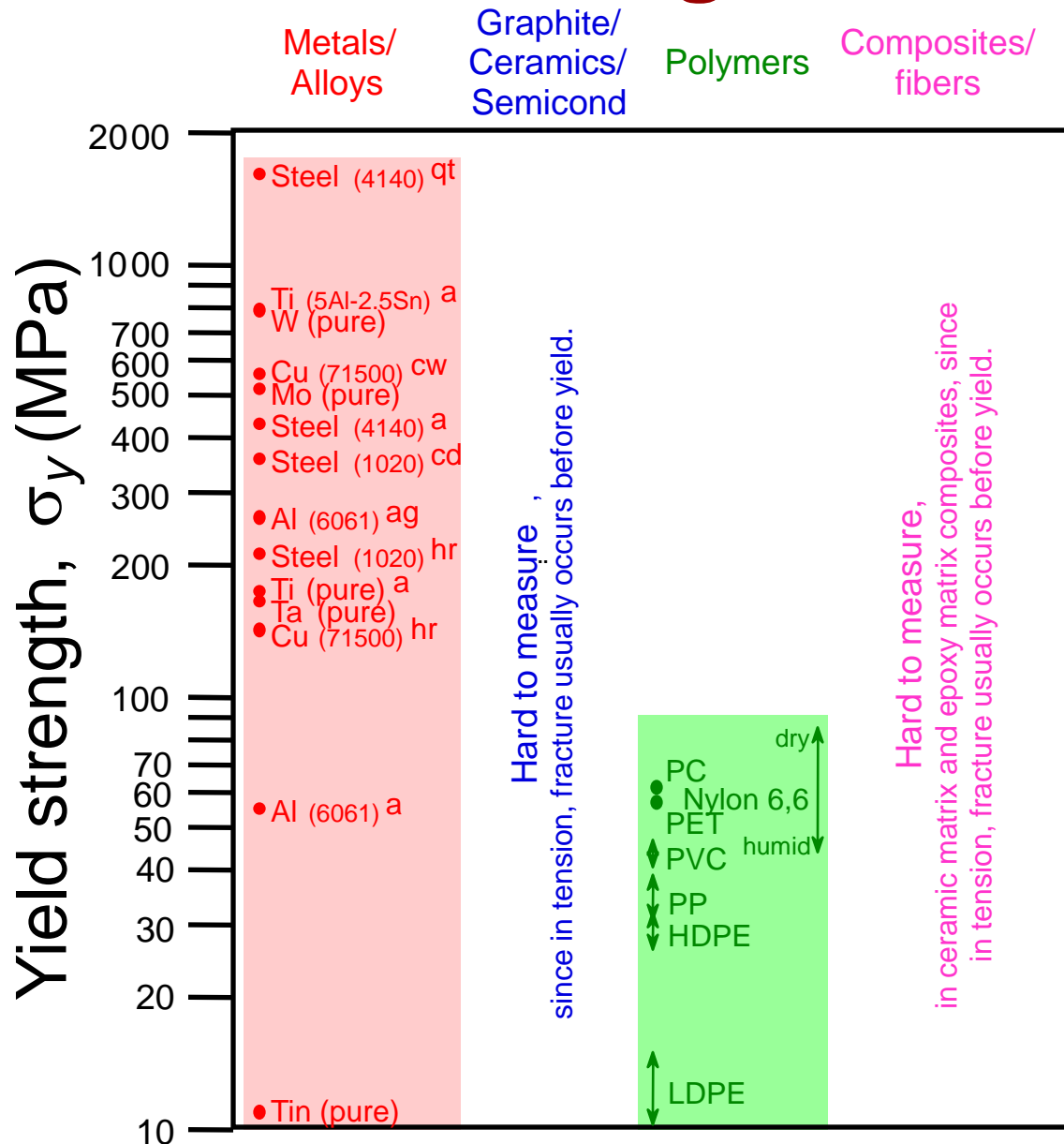
$$\varepsilon = 0.002 = \Delta z / z$$

$$\therefore \Delta z = 0.004 \text{ in}$$

Adapted from Fig. 6.10 (a),  
Callister 7e.



# Yield Strength : Comparison



## Room $T$ values

Based on data in Table B4,  
*Callister 7e*.

a = annealed

hr = hot rolled

ag = aged

cd = cold drawn

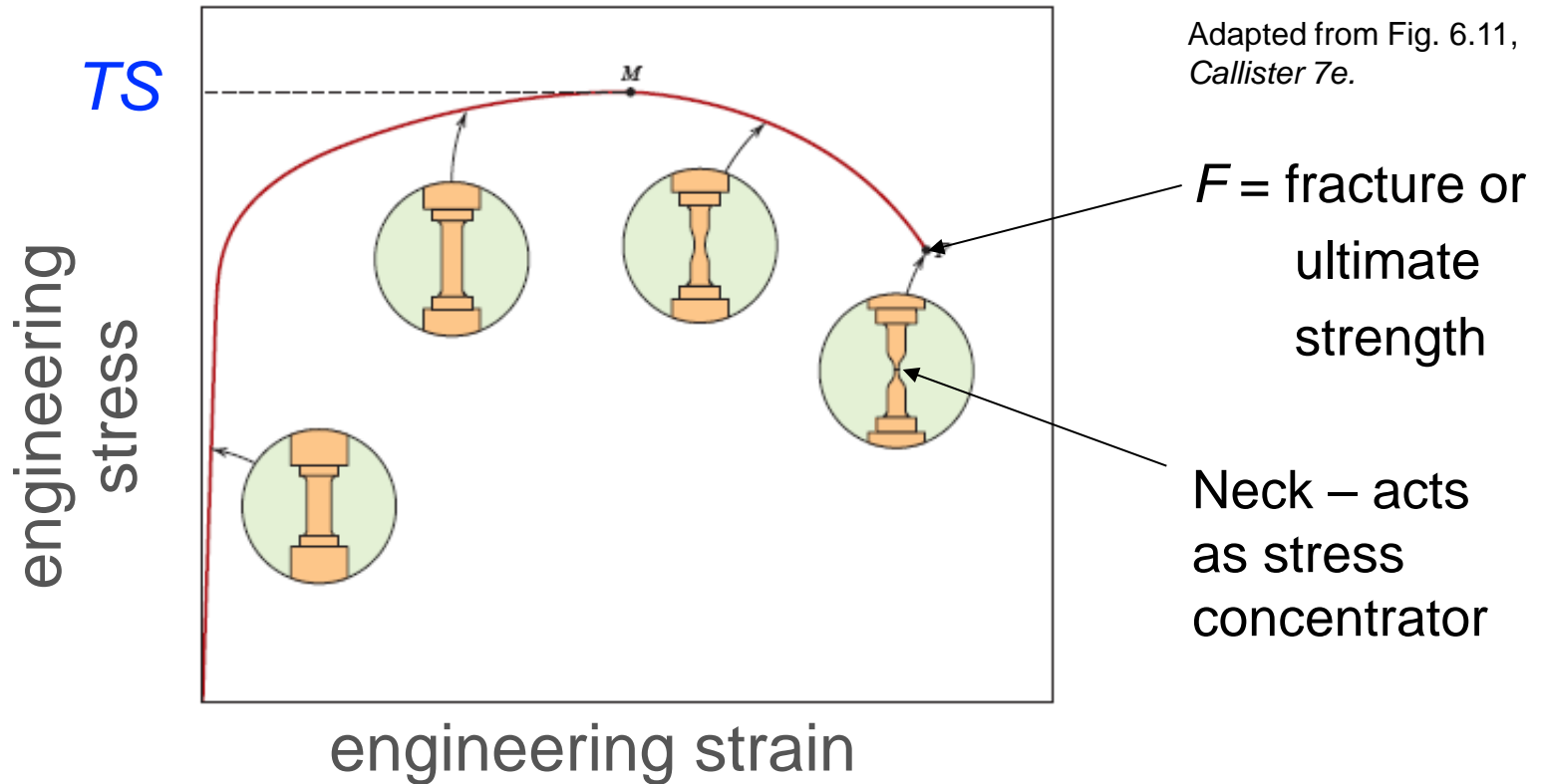
cw = cold worked

qt = quenched & tempered



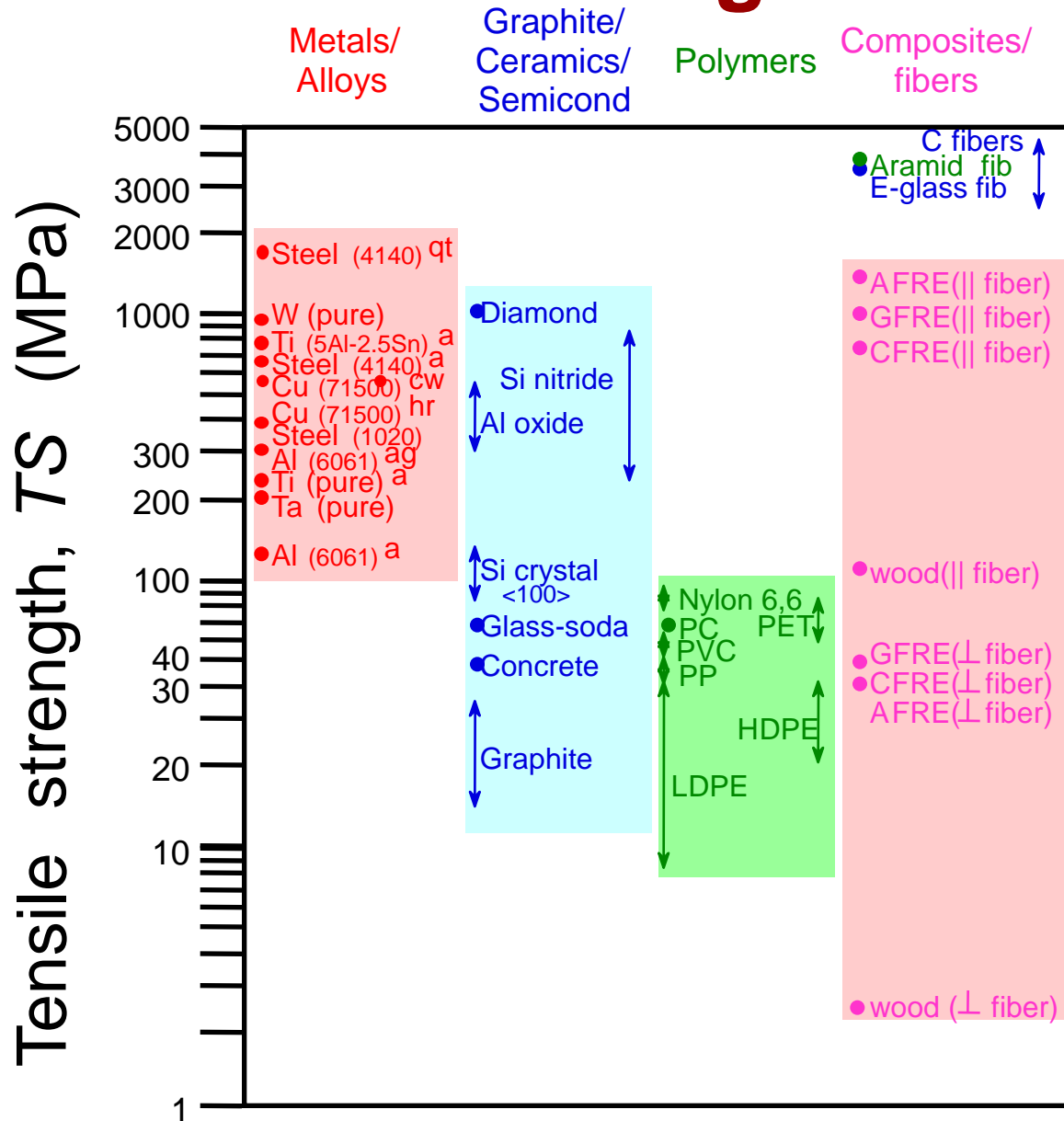
# Tensile Strength, TS

- Maximum stress on engineering stress-strain curve.



- Metals:** occurs when noticeable **necking** starts.
- Polymers:** occurs when **polymer backbone chains** are aligned and about to break.

# Tensile Strength : Comparison



## Room Temp. values

Based on data in Table B4,  
*Callister 7e.*

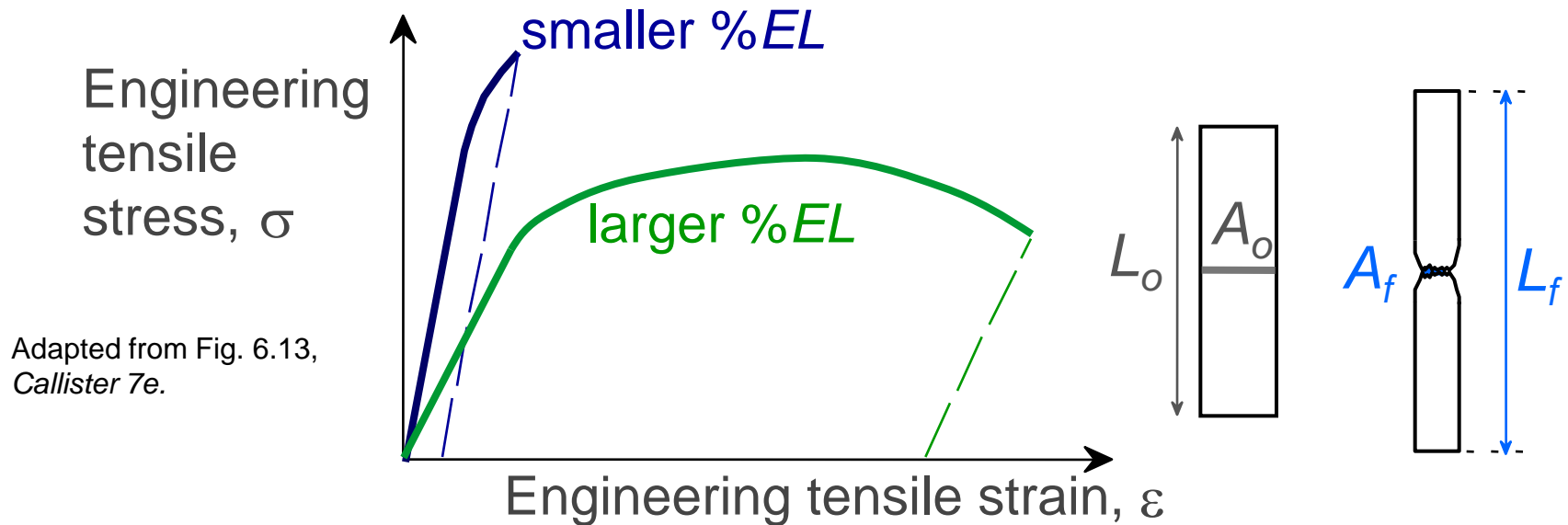
a = annealed  
hr = hot rolled  
ag = aged  
cd = cold drawn  
cw = cold worked  
qt = quenched & tempered  
AFRE, GFRE, & CFRE =  
aramid, glass, & carbon  
fiber-reinforced epoxy  
composites, with 60 vol%  
fibers.



# Ductility

- Plastic tensile strain at failure:

$$\%EL = \frac{L_f - L_o}{L_o} \times 100$$

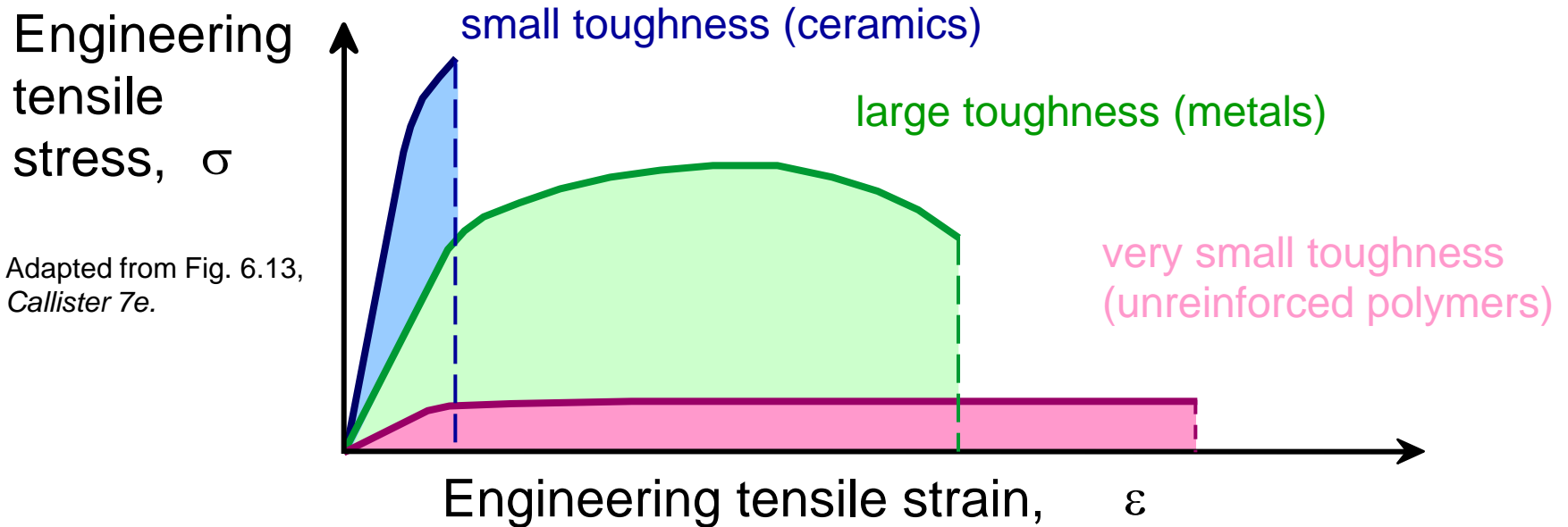


- Another ductility measure:

$$\%RA = \frac{A_o - A_f}{A_o} \times 100$$

# Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.



Adapted from Fig. 6.13,  
*Callister 7e*.

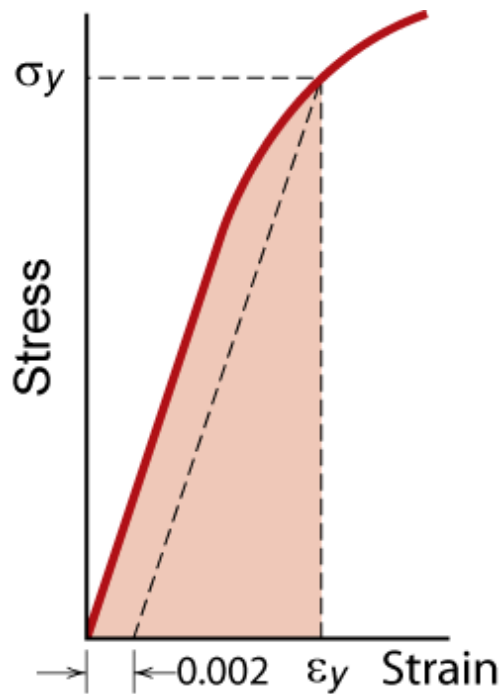
Brittle fracture: elastic energy

Ductile fracture: elastic + plastic energy



# Resilience, $U_r$

- Ability of a material to store energy
  - Energy stored best in elastic region



$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon$$

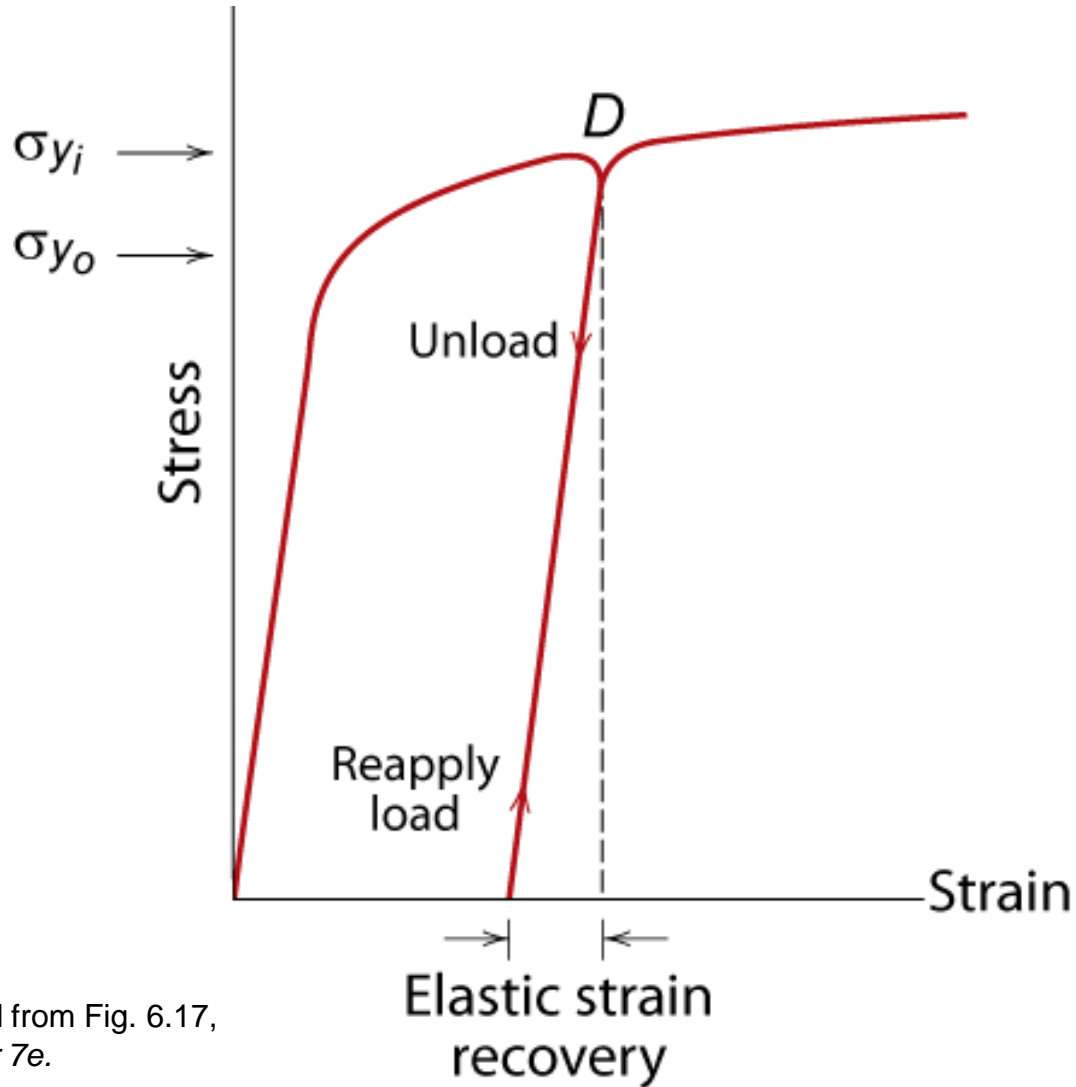
If we assume a linear stress-strain curve this simplifies to

$$U_r \cong \frac{1}{2} \sigma_y \epsilon_y$$

Adapted from Fig. 6.15,  
*Callister 7e.*



# Elastic Strain Recovery



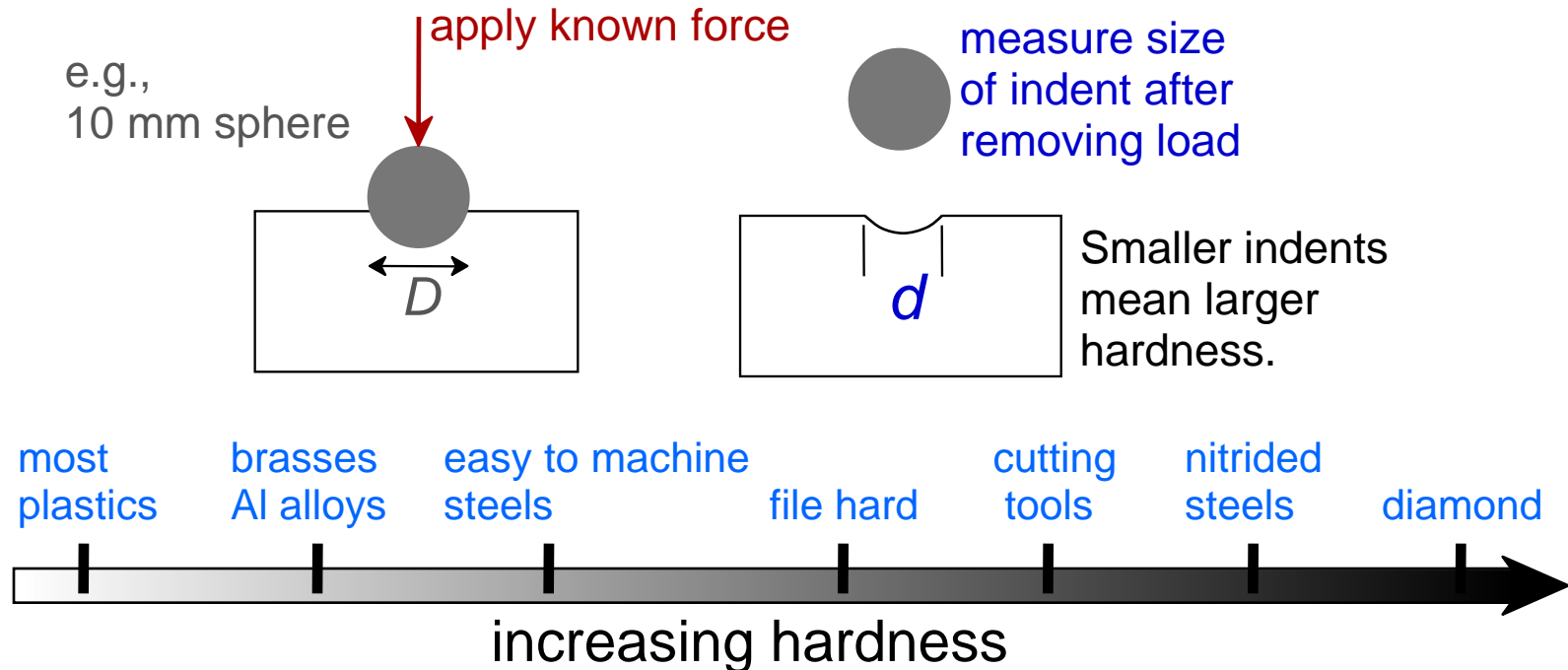
Adapted from Fig. 6.17,  
Callister 7e.





# Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression.
  - better wear properties.



# Hardness: Measurement

- Rockwell

- No major sample damage
- Each scale runs to 130 but only useful in range 20-100.
- Minor load    10 kg
- Major load    60 (A), 100 (B) & 150 (C) kg
  - A = diamond, B = 1/16 in. ball, C = diamond

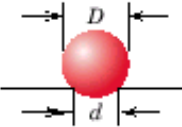
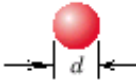
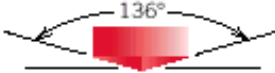

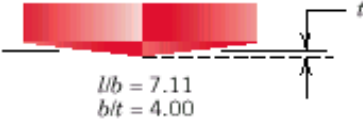
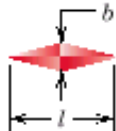
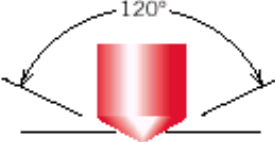



- HB = Brinell Hardness

- $TS \text{ (psia)} = 500 \times HB$
- $TS \text{ (MPa)} = 3.45 \times HB$



# Hardness: Measurement

Table 6.5 Hardness Testing Techniques

Test	Indenter	Shape of Indentation		Load	Formula for Hardness Number <sup>a</sup>
		Side View	Top View		
Brinell	10-mm sphere of steel or tungsten carbide			$P$	$HB = \frac{2P}{\pi D [D - \sqrt{D^2 - d^2}]}$
Vickers microhardness	Diamond pyramid			$P$	$HV = 1.854P/d_1^2$
Knoop microhardness	Diamond pyramid			$P$	$HK = 14.2P/l^2$
Rockwell and Superficial Rockwell	<div>           Diamond cone  <math>\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}</math> in. diameter            steel spheres         </div>	  	  	<div>           60 kg            100 kg            150 kg         </div> } Rockwell <div>           15 kg            30 kg            45 kg         </div> } Superficial Rockwell	

<sup>a</sup> For the hardness formulas given,  $P$  (the applied load) is in kg, while  $D$ ,  $d$ ,  $d_1$ , and  $l$  are all in mm.

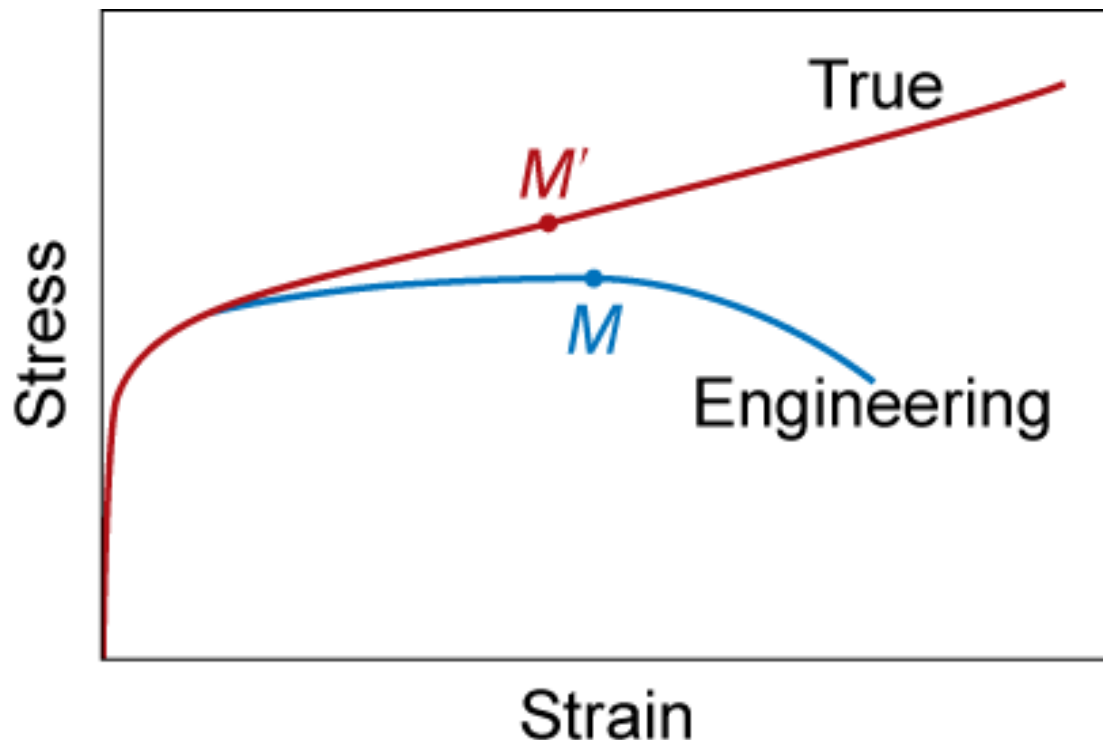
**Source:** Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

# True Stress & Strain

Note: S.A. changes when sample stretched

- True stress  $\sigma_T = F/A_i$
- True Strain  $\epsilon_T = \ln(\ell_i/\ell_o)$

$$\sigma_T = \sigma(1 + \epsilon)$$
$$\epsilon_T = \ln(1 + \epsilon)$$

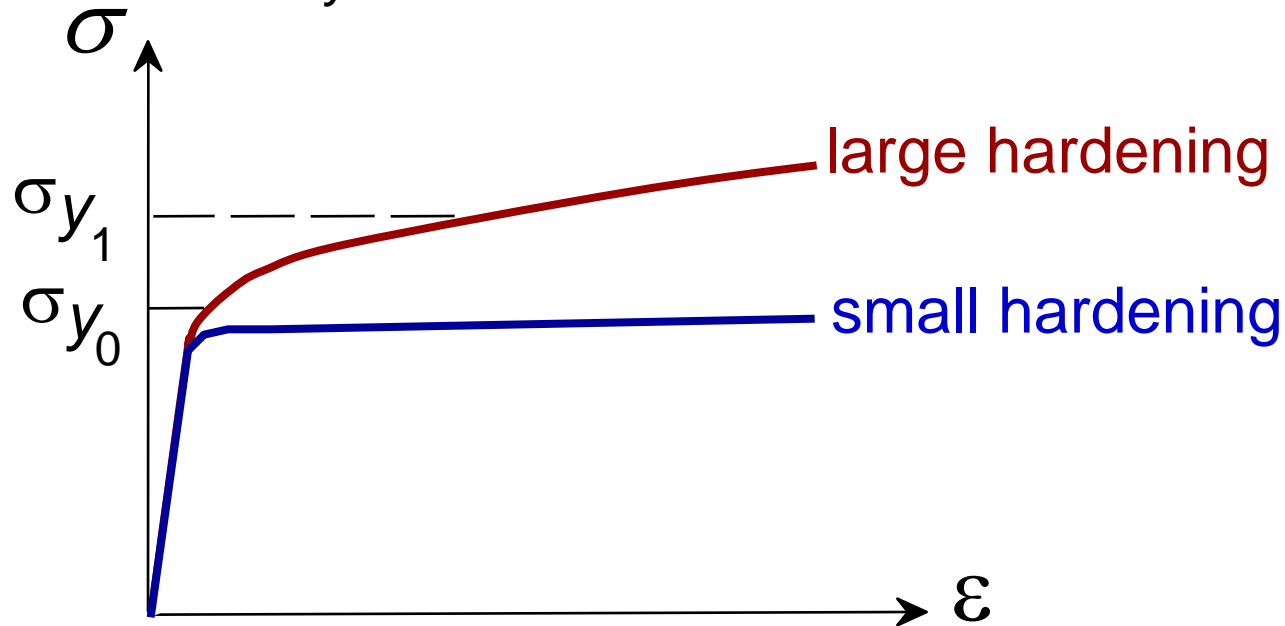


Adapted from Fig. 6.16,  
*Callister 7e.*



# Hardening

- An increase in  $\sigma_y$  due to plastic deformation.



- Curve fit to the stress-strain response:

$$\sigma_T = K(\epsilon_T)^n$$

“true” stress ( $F/A$ )

“true” strain:  $\ln(L/L_0)$

hardening exponent:  
 $n = 0.15$  (some steels)  
to  $n = 0.5$  (some coppers)



# Variability in Material Properties

- Elastic modulus is material property
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics

– Mean

$$\bar{x} = \frac{\sum^n x_n}{n}$$

– Standard Deviation

$$s = \left[ \frac{\sum^n (x_i - \bar{x})^2}{n-1} \right]^{\frac{1}{2}}$$

where  $n$  is the number of data points



# Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety,  $N$

$$\sigma_{working} = \frac{\sigma_y}{N}$$

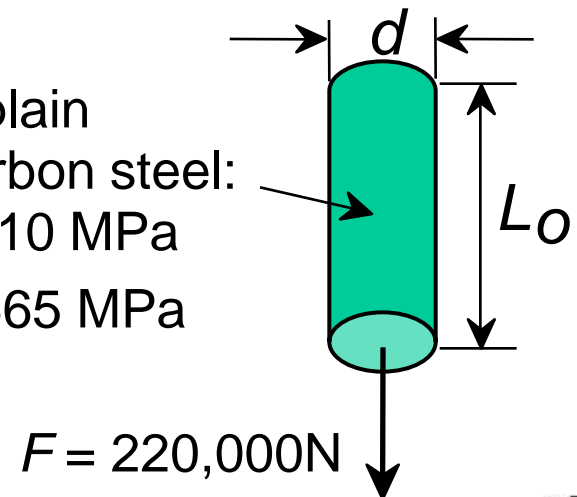
Often  $N$  is  
between  
1.2 and 4

- Example: Calculate a diameter,  $d$ , to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

$$\frac{220,000N}{\pi(d^2 / 4)} = \frac{\sigma_y}{5}$$

$$d = 0.067 \text{ m} = 6.7 \text{ cm}$$

1045 plain  
carbon steel:  
 $\sigma_y = 310 \text{ MPa}$   
 $TS = 565 \text{ MPa}$



# Summary

- **Stress** and **strain**: These are size-independent measures of load and displacement, respectively.
- **Elastic** behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus ( $E$  or  $G$ ).
- **Plastic** behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches  $\sigma_y$ .
- **Toughness**: The energy needed to break a unit volume of material.
- **Ductility**: The plastic strain at failure.





# ANNOUNCEMENTS

Reading:

Core Problems:

Self-help Problems:

