

# Chapter 9: Phase Diagrams

## ISSUES TO ADDRESS...

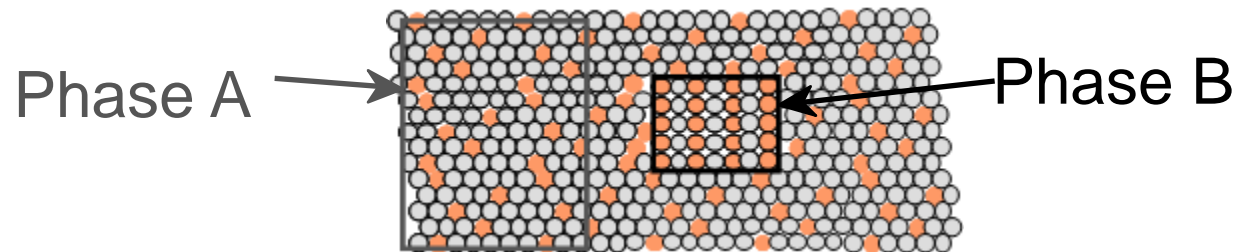
- When we combine two elements...  
    what equilibrium state do we get?
- In particular, if we specify...
  - a composition (e.g., wt% Cu - wt% Ni), and
  - a temperature ( $T$ )

then...

How many phases do we get?

What is the composition of each phase?

How much of each phase do we get?



- Nickel atom
- Copper atom

# Phase Equilibria: Solubility Limit

## Introduction

- **Solutions** – solid solutions, single phase
- **Mixtures** – more than one phase

Adapted from Fig. 9.1,  
Callister 7e.

- **Solubility Limit:**

Max concentration for which only a single phase solution occurs.

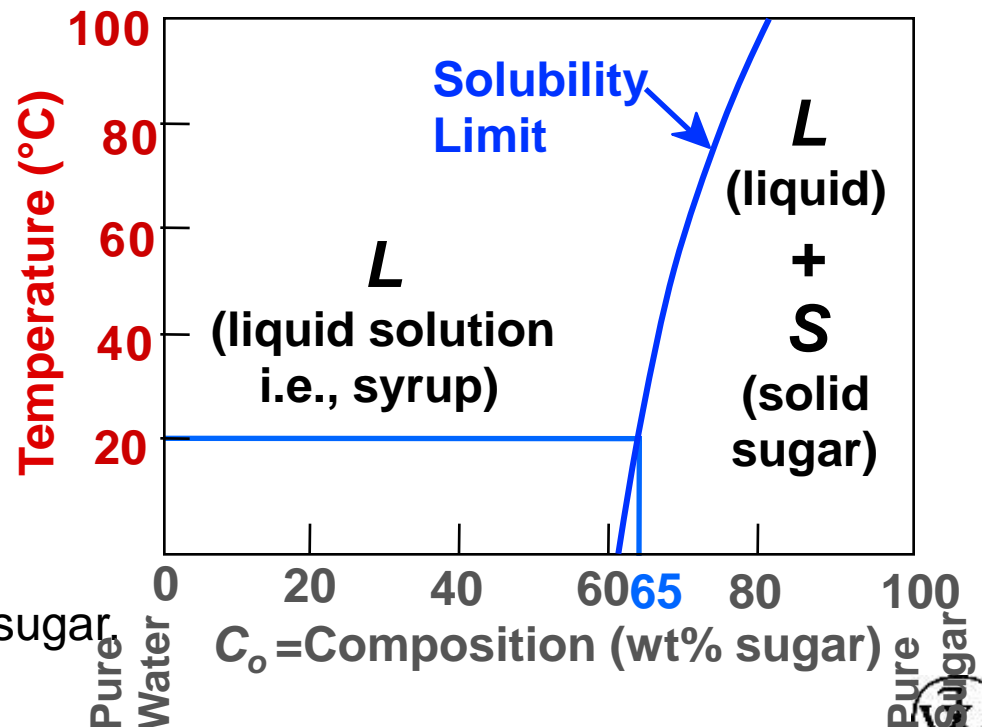
Question: What is the solubility limit at **20°C**?

Answer: **65 wt% sugar**.

If  $C_0 < 65$  wt% sugar: syrup

If  $C_0 > 65$  wt% sugar: syrup + sugar

Sucrose/Water Phase Diagram

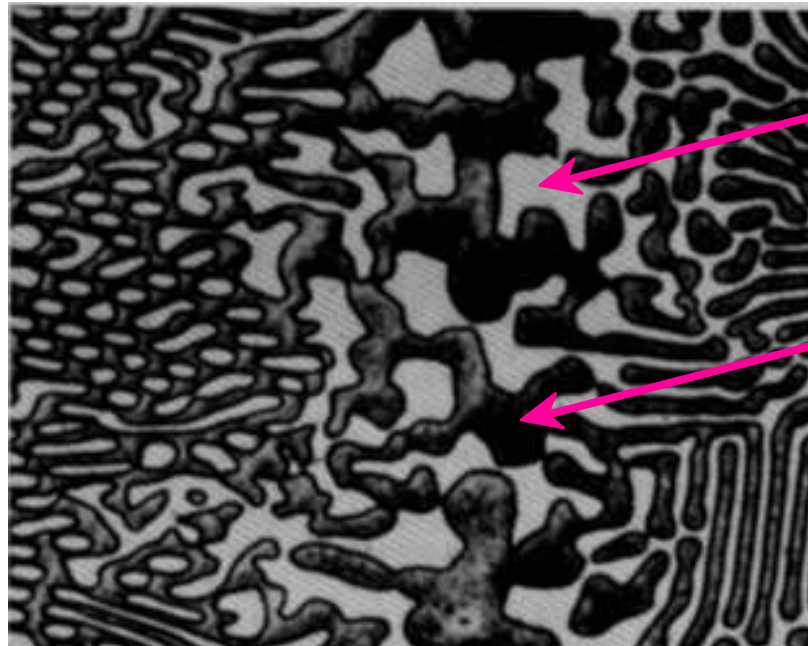


# Components and Phases

- **Components:**  
The elements or compounds which are present in the mixture (e.g., Al and Cu)
- **Phases:**  
The physically and chemically distinct material regions that result (e.g.,  $\alpha$  and  $\beta$ ).

Aluminum-  
Copper  
Alloy

Adapted from  
chapter-opening  
photograph,  
Chapter 9,  
*Callister 3e*.

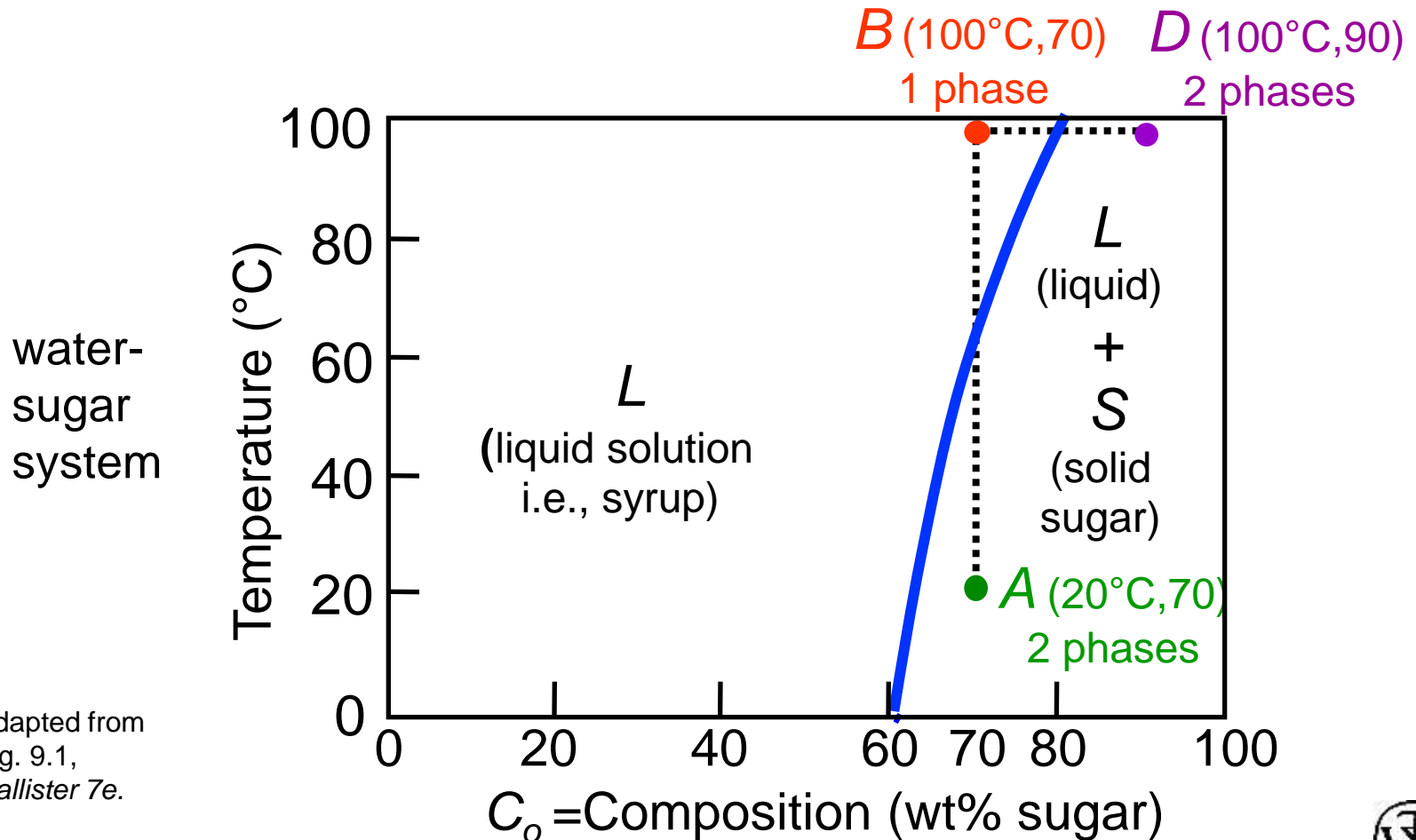


$\beta$  (lighter  
phase)

$\alpha$  (darker  
phase)

# Effect of $T$ & Composition ( $C_o$ )

- Changing  $T$  can change # of phases: path  $A$  to  $B$ .
- Changing  $C_o$  can change # of phases: path  $B$  to  $D$ .



Adapted from  
Fig. 9.1,  
Callister 7e.



# Phase Equilibria

Simple solution system (e.g., Ni-Cu solution)

	Crystal Structure	electroneg	$r$ (nm)
Ni	FCC	1.9	0.1246
Cu	FCC	1.8	0.1278

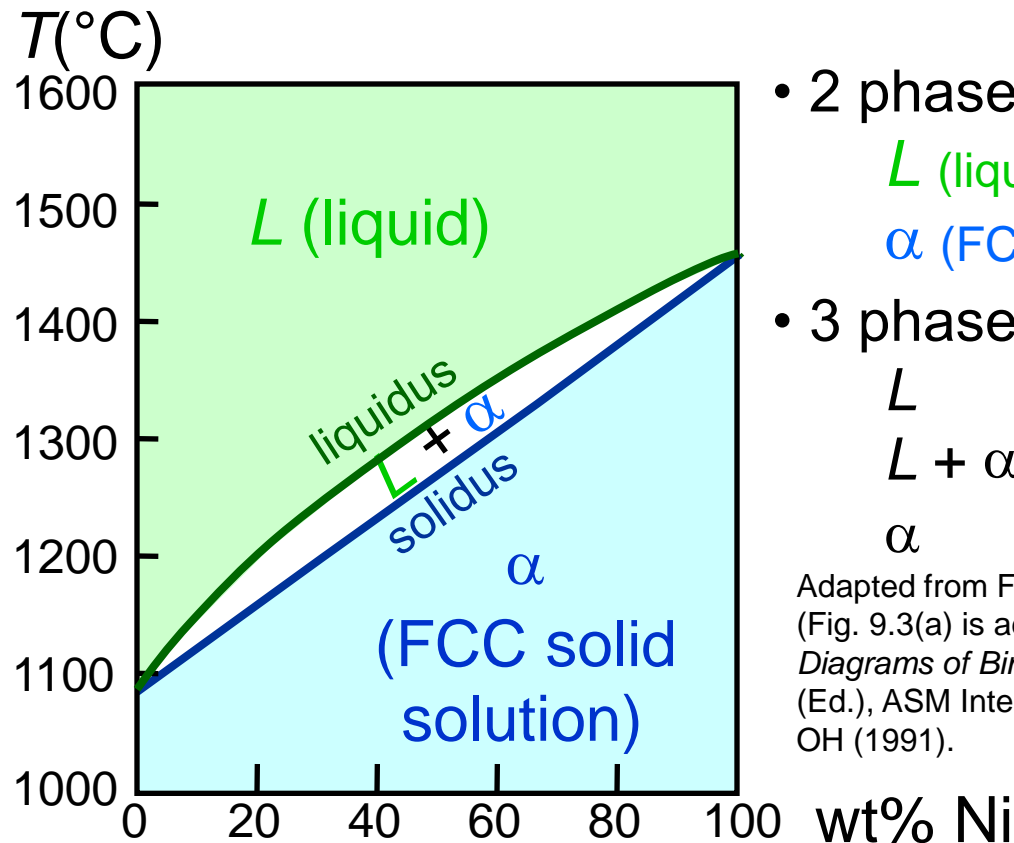
- Both have the same crystal structure (FCC) and have similar electronegativities and atomic radii (W. Hume – Rothery rules) suggesting high mutual solubility.
- Ni and Cu are totally miscible in all proportions.



# Phase Diagrams

- Indicate phases as function of  $T$ ,  $C_o$ , and  $P$ .
- For this course:
  - binary systems: just 2 components.
  - independent variables:  $T$  and  $C_o$  ( $P = 1$  atm is almost always used).

- Phase Diagram for Cu-Ni system



- 2 phases:
  - $L$  (liquid)
  - $\alpha$  (FCC solid solution)
- 3 phase fields:
  - $L$
  - $L + \alpha$
  - $\alpha$

Adapted from Fig. 9.3(a), Callister 7e.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH (1991).



# Phase Diagrams: # and types of phases

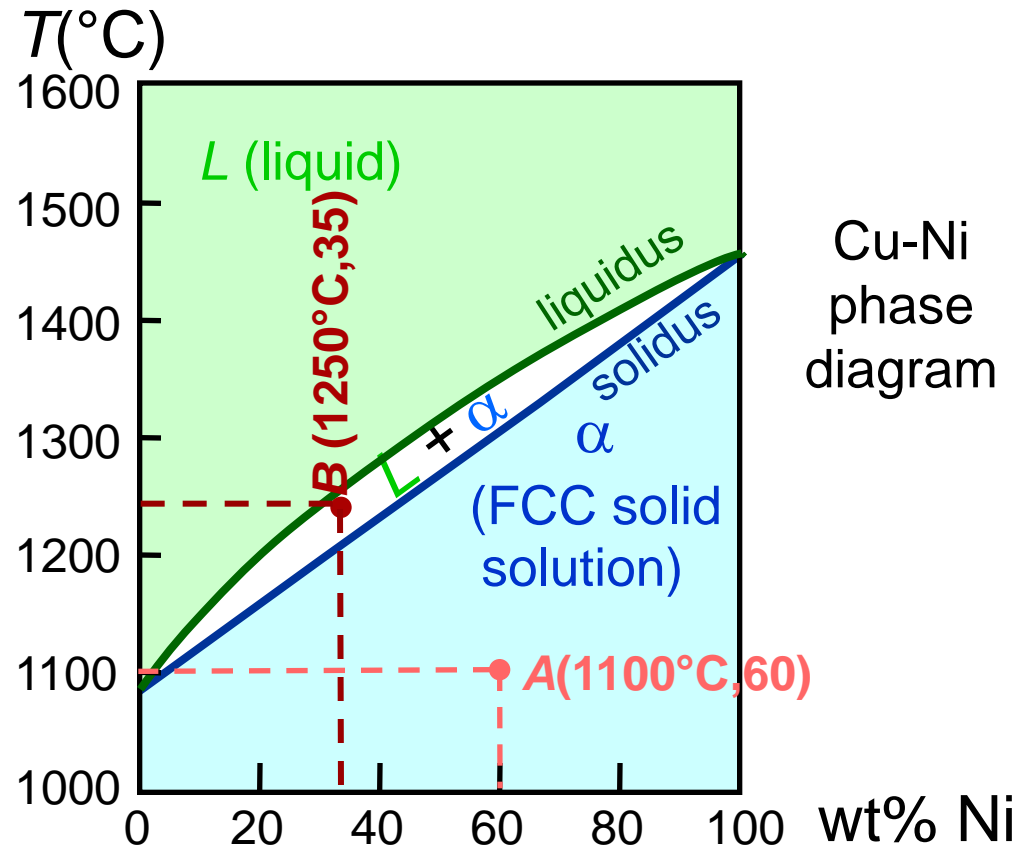
- Rule 1: If we know  $T$  and  $C_o$ , then we know:  
--the # and types of phases present.

- Examples:

$A(1100^{\circ}\text{C}, 60)$ :  
1 phase:  $\alpha$

$B(1250^{\circ}\text{C}, 35)$ :  
2 phases:  $L + \alpha$

Adapted from Fig. 9.3(a), *Callister 7e*.  
(Fig. 9.3(a) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991).



# Phase Diagrams: composition of phases

- Rule 2: If we know  $T$  and  $C_0$ , then we know:  
--the composition of each phase.

- Examples:

$C_0 = 35 \text{ wt\% Ni}$

At  $T_A = 1320^\circ\text{C}$ :

Only Liquid ( $L$ )

$C_L = C_0 (= 35 \text{ wt\% Ni})$

At  $T_D = 1190^\circ\text{C}$ :

Only Solid ( $\alpha$ )

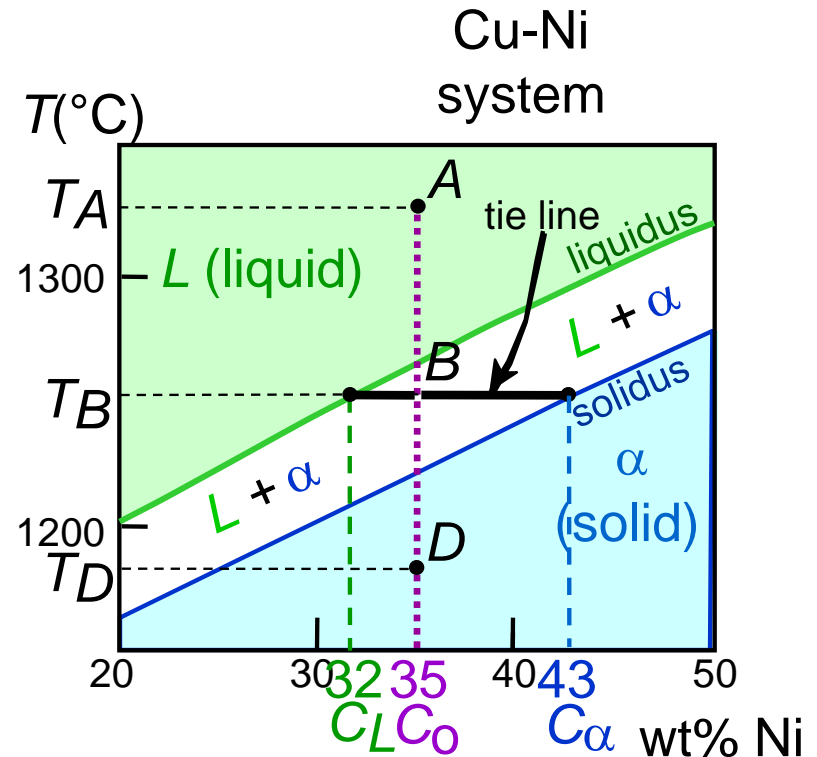
$C_\alpha = C_0 (= 35 \text{ wt\% Ni})$

At  $T_B = 1250^\circ\text{C}$ :

Both  $\alpha$  and  $L$

$C_L = C_{\text{liquidus}} (= 32 \text{ wt\% Ni here})$

$C_\alpha = C_{\text{solidus}} (= 43 \text{ wt\% Ni here})$



Adapted from Fig. 9.3(b), Callister 7e.  
(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)





# Phase Diagrams: weight fractions of phases

- Rule 3: If we know  $T$  and  $C_0$ , then we know:  
--the amount of each phase (given in wt%).

- Examples:

$C_0 = 35 \text{ wt\% Ni}$

At  $T_A$ : Only Liquid (L)

$W_L = 100 \text{ wt\%}, W_\alpha = 0$

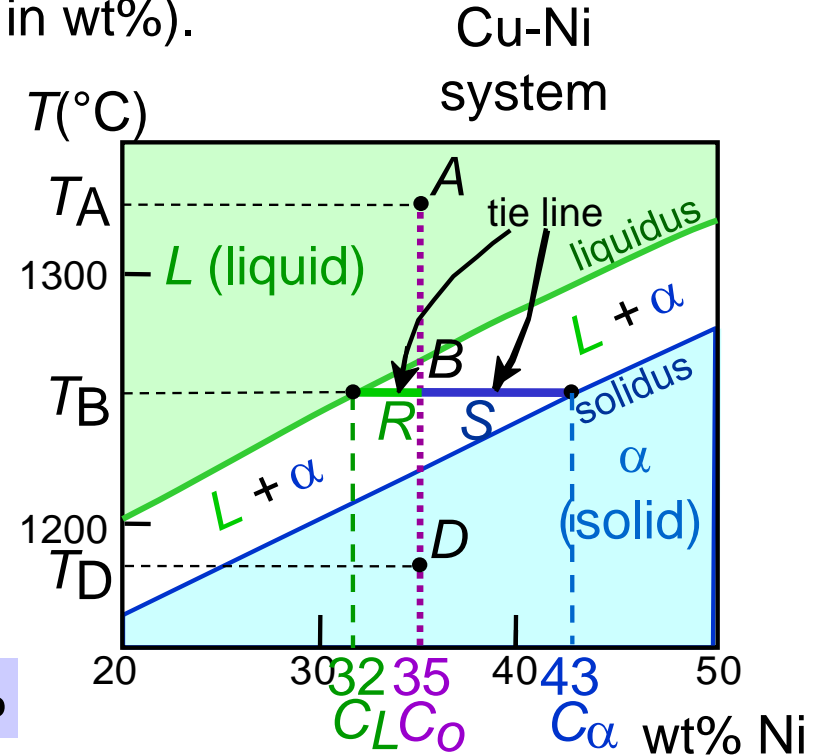
At  $T_D$ : Only Solid ( $\alpha$ )

$W_L = 0, W_\alpha = 100 \text{ wt\%}$

At  $T_B$ : Both  $\alpha$  and L

$$W_L = \frac{S}{R+S} = \frac{43-35}{43-32} = 73 \text{ wt\%}$$

$$W_\alpha = \frac{R}{R+S} = 27 \text{ wt\%}$$



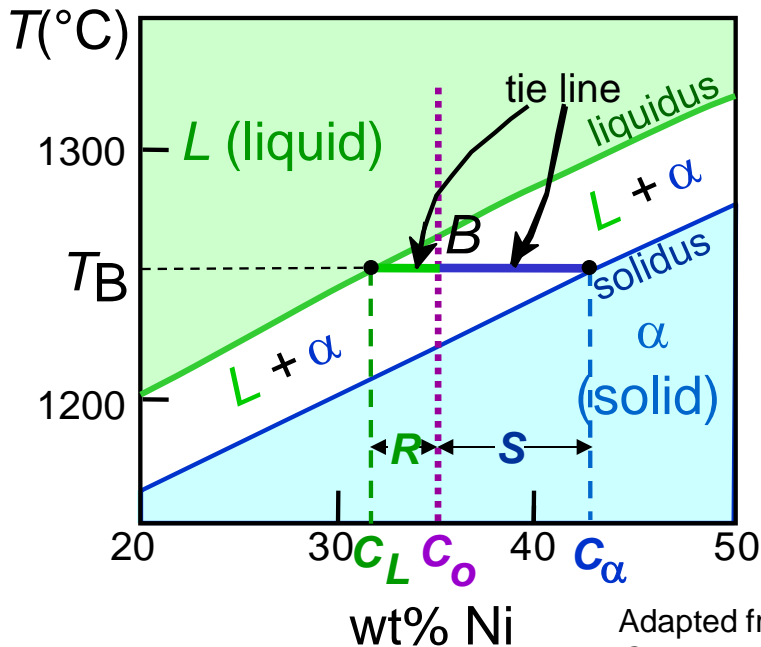
Adapted from Fig. 9.3(b), Callister 7e.

(Fig. 9.3(b) is adapted from *Phase Diagrams of Binary Nickel Alloys*, P. Nash (Ed.), ASM International, Materials Park, OH, 1991.)



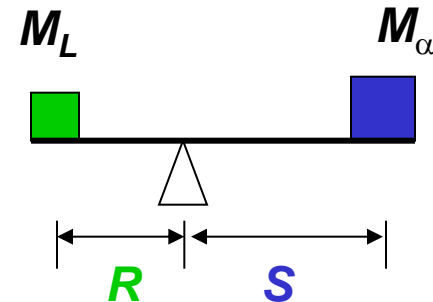
# The Lever Rule

- Tie line – connects the phases in equilibrium with each other - essentially an isotherm



How much of each phase?

Think of it as a lever (teeter-totter)



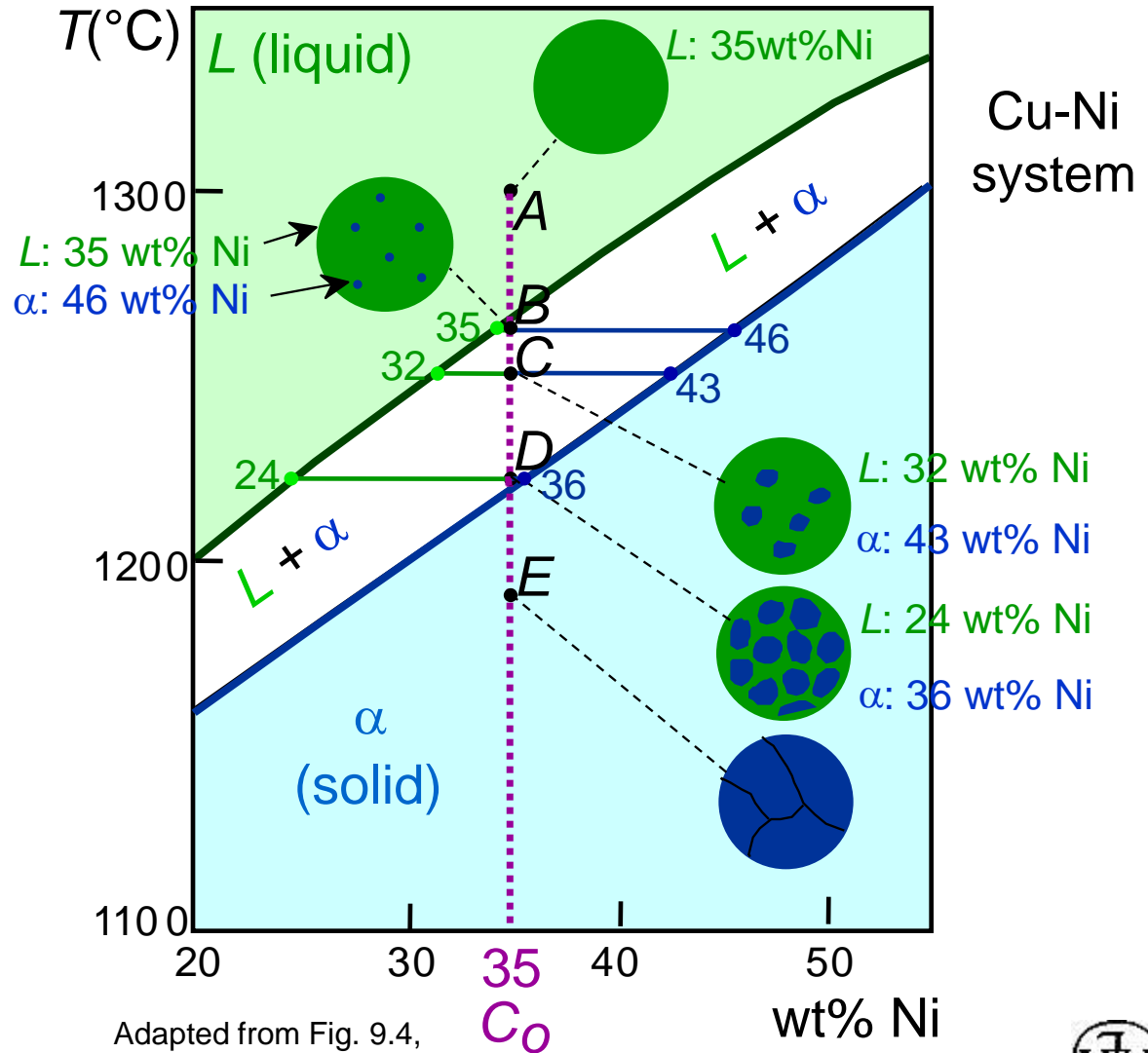
$$M_{\alpha} \cdot S = M_L \cdot R$$

$$W_L = \frac{M_L}{M_L + M_{\alpha}} = \frac{S}{R + S} = \frac{C_{\alpha} - C_0}{C_{\alpha} - C_L}$$

$$W_{\alpha} = \frac{R}{R + S} = \frac{C_0 - C_L}{C_{\alpha} - C_L}$$

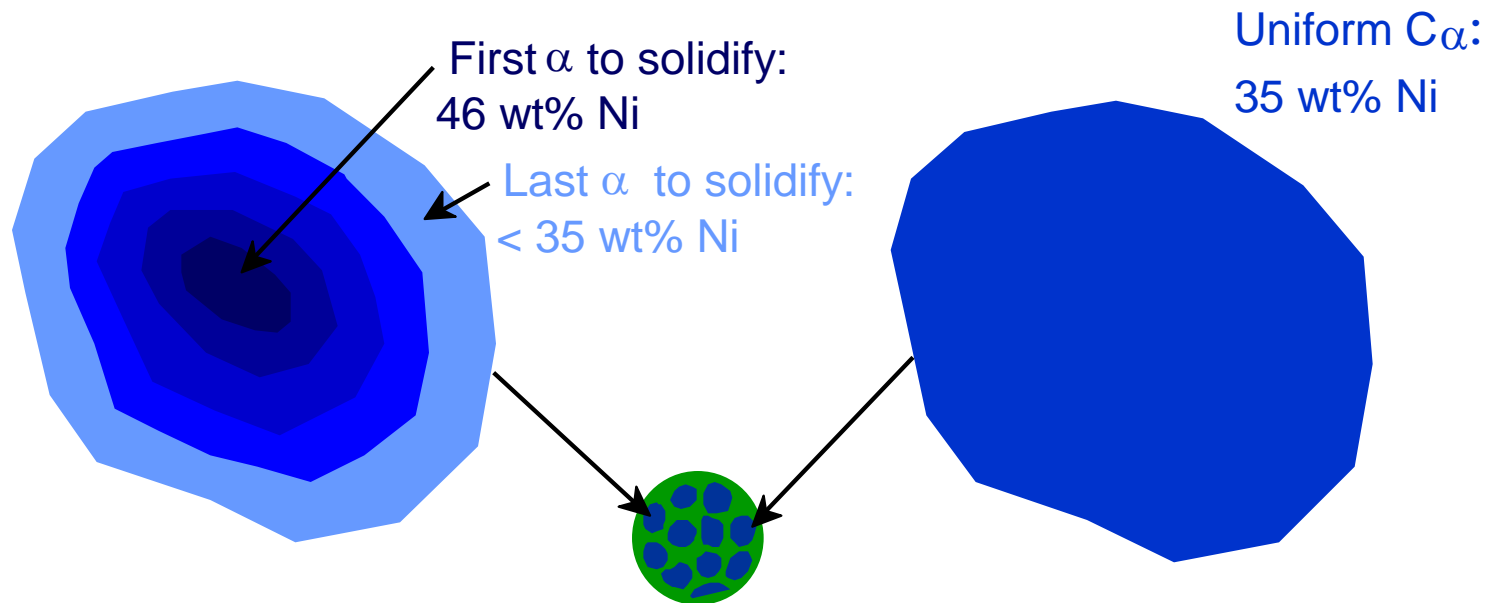
# Ex: Cooling in a Cu-Ni Binary

- Phase diagram:  
Cu-Ni system.
- System is:
  - binary  
i.e., 2 components:  
Cu and Ni.
  - isomorphous  
i.e., complete  
solubility of one  
component in  
another;  $\alpha$  phase  
field extends from  
0 to 100 wt% Ni.
- Consider  
 $C_0 = 35 \text{ wt\% Ni}$ .



# Cored vs Equilibrium Phases

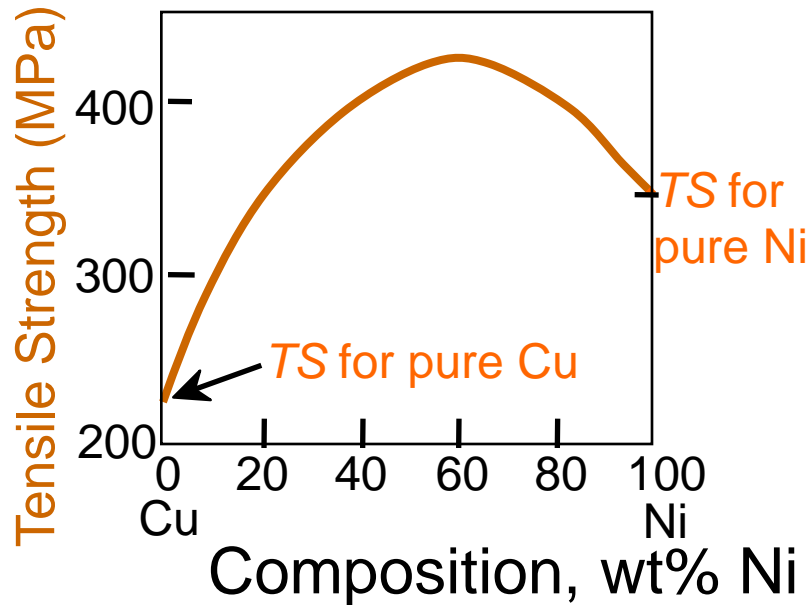
- $C_\alpha$  changes as we solidify.
- Cu-Ni case: First  $\alpha$  to solidify has  $C_\alpha = 46$  wt% Ni.  
Last  $\alpha$  to solidify has  $C_\alpha = 35$  wt% Ni.
- Fast rate of cooling:  
Cored structure
- Slow rate of cooling:  
Equilibrium structure



# Mechanical Properties: Cu-Ni System

- Effect of solid solution strengthening on:

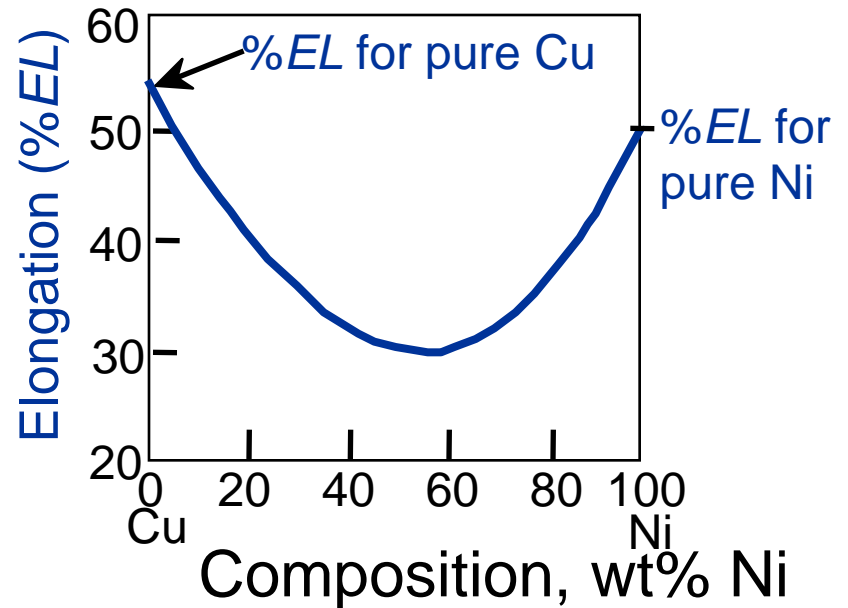
--Tensile strength ( $TS$ )



Adapted from Fig. 9.6(a), Callister 7e.

--Peak as a function of  $C_0$

--Ductility ( $\%EL$ ,  $\%AR$ )



Adapted from Fig. 9.6(b), Callister 7e.

--Min. as a function of  $C_0$



# Binary-Eutectic Systems

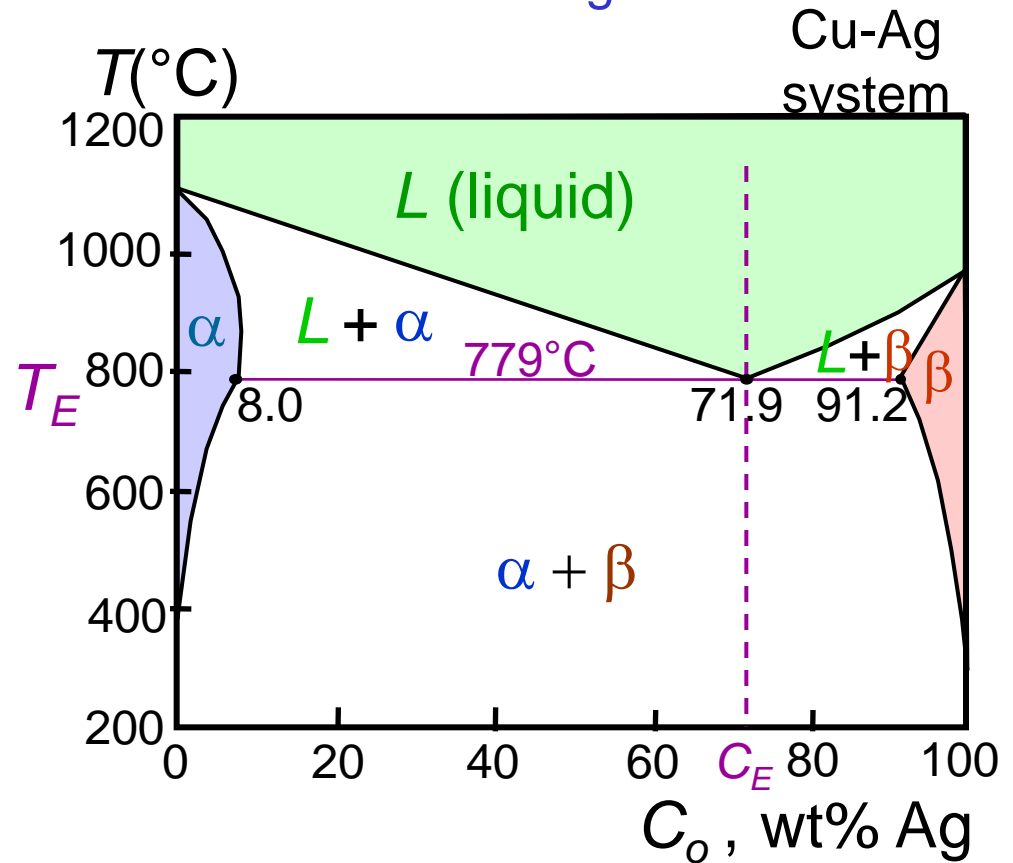
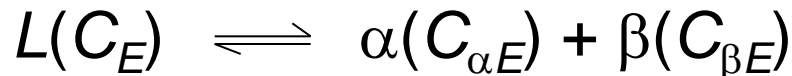
2 components

has a special composition with a min. melting T.

Ex.: Cu-Ag system

- 3 single phase regions ( $L$ ,  $\alpha$ ,  $\beta$ )
- Limited solubility:  
 $\alpha$ : mostly Cu  
 $\beta$ : mostly Ag
- $T_E$ : No liquid below  $T_E$
- $C_E$ : Min. melting  $T_E$  composition

## Eutectic transition



Adapted from Fig. 9.7,  
Callister 7e.



# EX: Pb-Sn Eutectic System (1)

- For a 40 wt% Sn-60 wt% Pb alloy at 150°C, find...

--the phases present:  $\alpha + \beta$

--compositions of phases:

$$C_o = 40 \text{ wt\% Sn}$$

$$C_\alpha = 11 \text{ wt\% Sn}$$

$$C_\beta = 99 \text{ wt\% Sn}$$

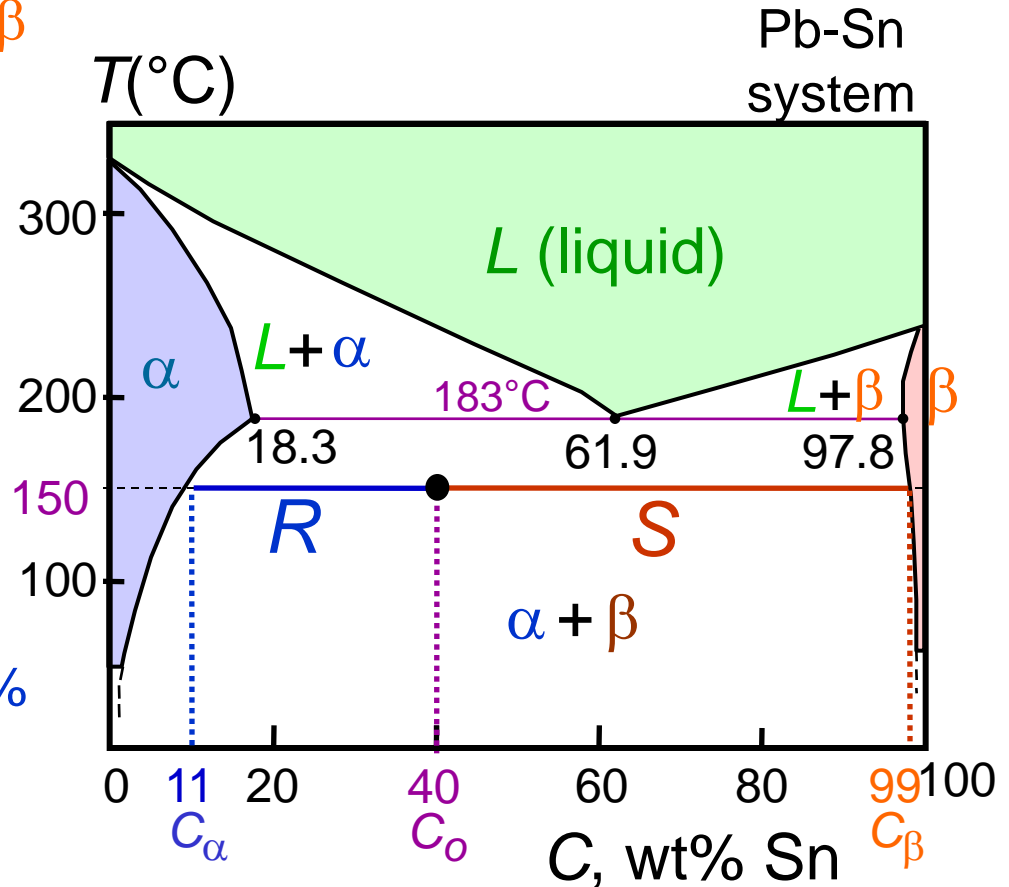
--the relative amount of each phase:

$$W_\alpha = \frac{S}{R+S} = \frac{C_\beta - C_o}{C_\beta - C_\alpha}$$

$$= \frac{99 - 40}{99 - 11} = \frac{59}{88} = 67 \text{ wt\%}$$

$$W_\beta = \frac{R}{R+S} = \frac{C_o - C_\alpha}{C_\beta - C_\alpha}$$

$$= \frac{40 - 11}{99 - 11} = \frac{29}{88} = 33 \text{ wt\%}$$



# EX: Pb-Sn Eutectic System (2)

- For a 40 wt% Sn-60 wt% Pb alloy at 200°C, find...

--the phases present:  $\alpha + L$

--compositions of phases:

$$C_o = 40 \text{ wt\% Sn}$$

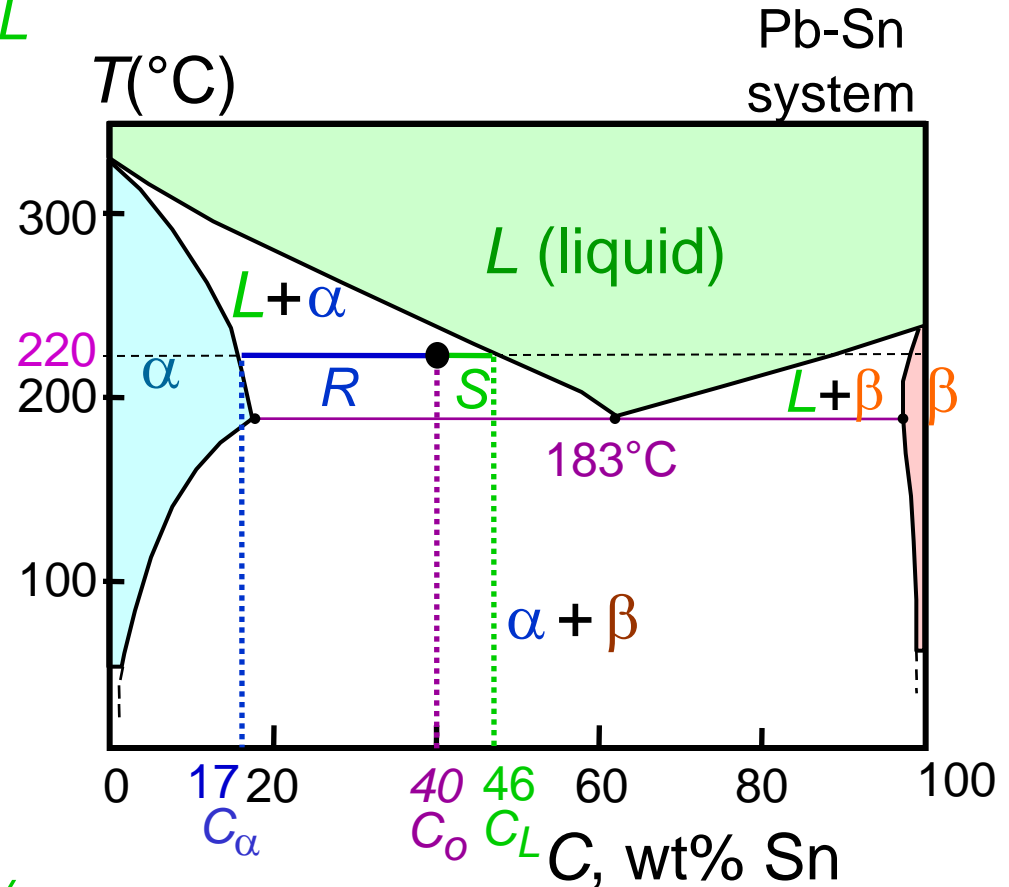
$$C_\alpha = 17 \text{ wt\% Sn}$$

$$C_L = 46 \text{ wt\% Sn}$$

--the relative amount of each phase:

$$W_\alpha = \frac{C_L - C_o}{C_L - C_\alpha} = \frac{46 - 40}{46 - 17} = \frac{6}{29} = 21 \text{ wt\%}$$

$$W_L = \frac{C_o - C_\alpha}{C_L - C_\alpha} = \frac{23}{29} = 79 \text{ wt\%}$$



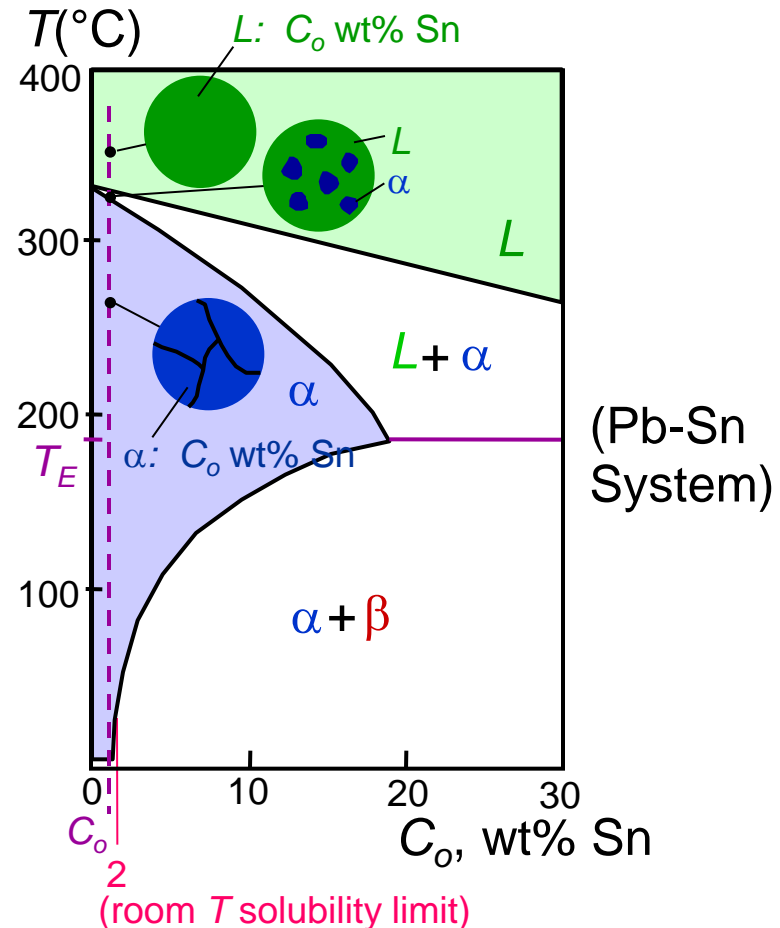
Adapted from Fig. 9.8,  
Callister 7e.





# Microstructures in Eutectic Systems: I

- $C_o < 2 \text{ wt\% Sn}$
- Result:
  - at extreme ends
  - polycrystal of  $\alpha$  grains  
i.e., only one solid phase.

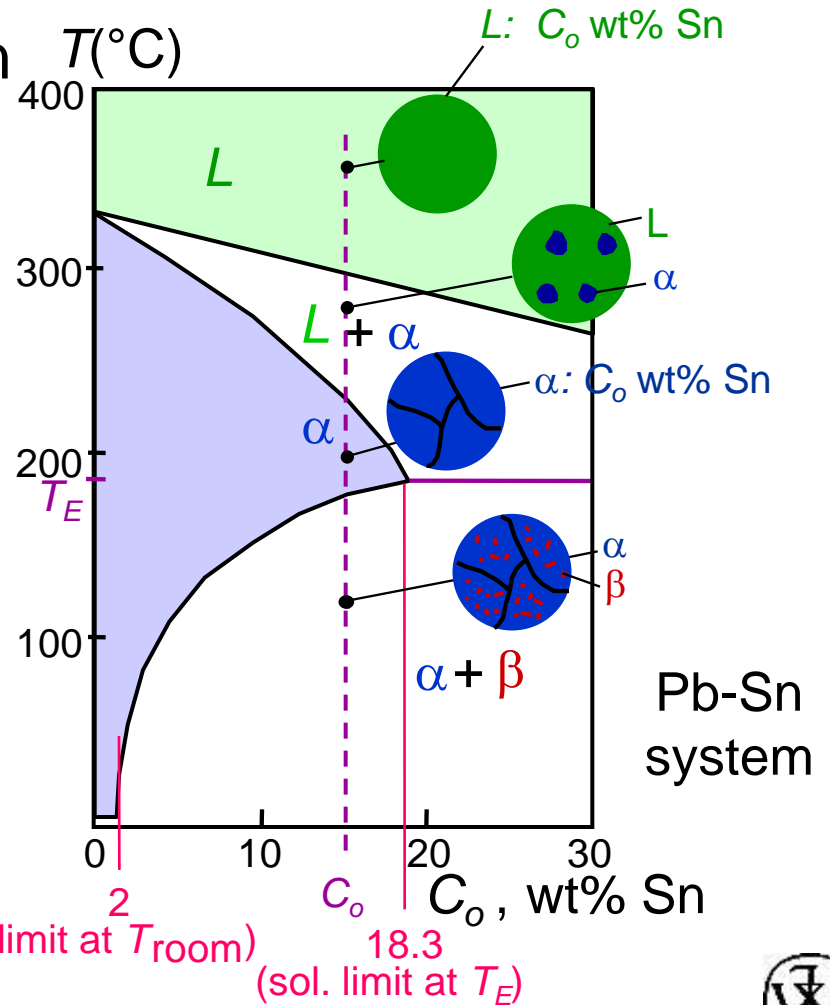


Adapted from Fig. 9.11,  
Callister 7e.



# Microstructures in Eutectic Systems: II

- 2 wt% Sn <  $C_0$  < 18.3 wt% Sn
- Result:
  - Initially liquid +  $\alpha$
  - then  $\alpha$  alone
  - finally two phases
    - $\alpha$  polycrystal
    - fine  $\beta$ -phase inclusions



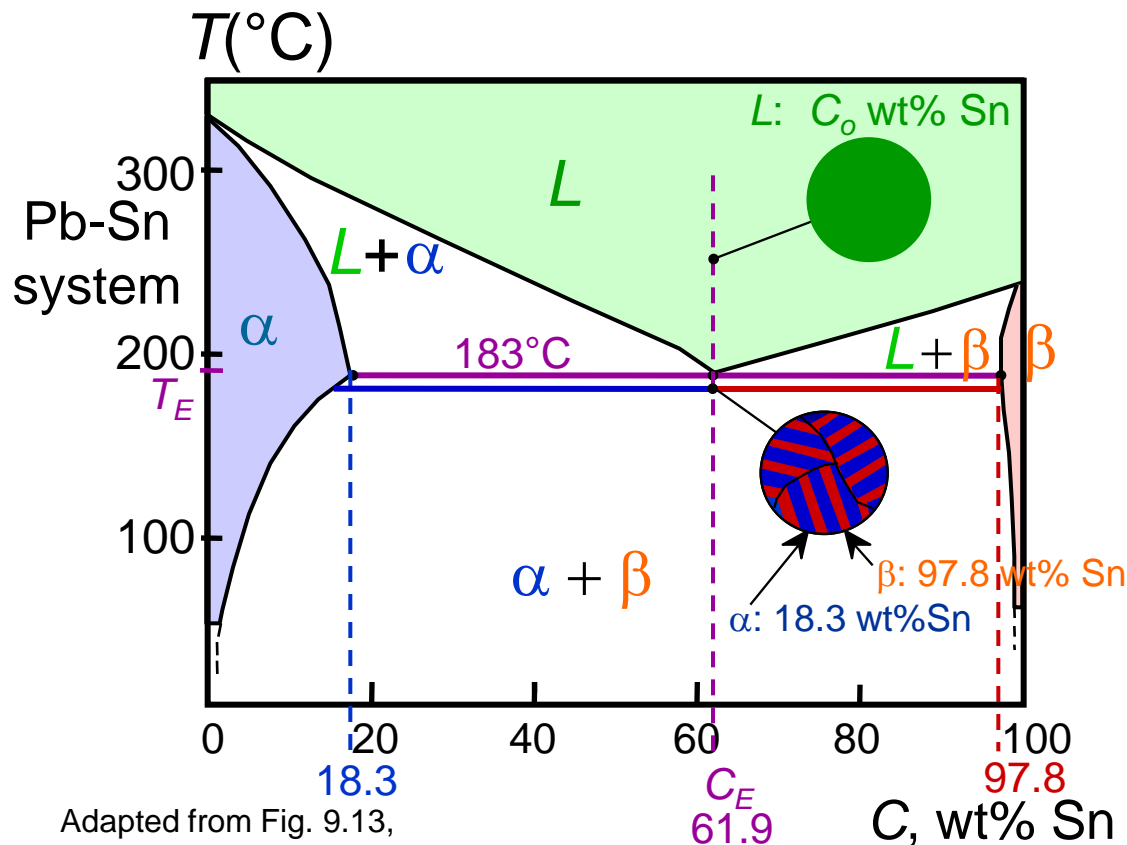
Adapted from Fig. 9.12,  
Callister 7e.

(sol. limit at  $T_{\text{room}}$ )  
2  
 $C_0$   
18.3  
(sol. limit at  $T_E$ )



# Microstructures in Eutectic Systems: III

- $C_0 = C_E$
- Result: Eutectic microstructure (lamellar structure)  
--alternating layers (lamellae) of  $\alpha$  and  $\beta$  crystals.



Adapted from Fig. 9.13,  
Callister 7e.

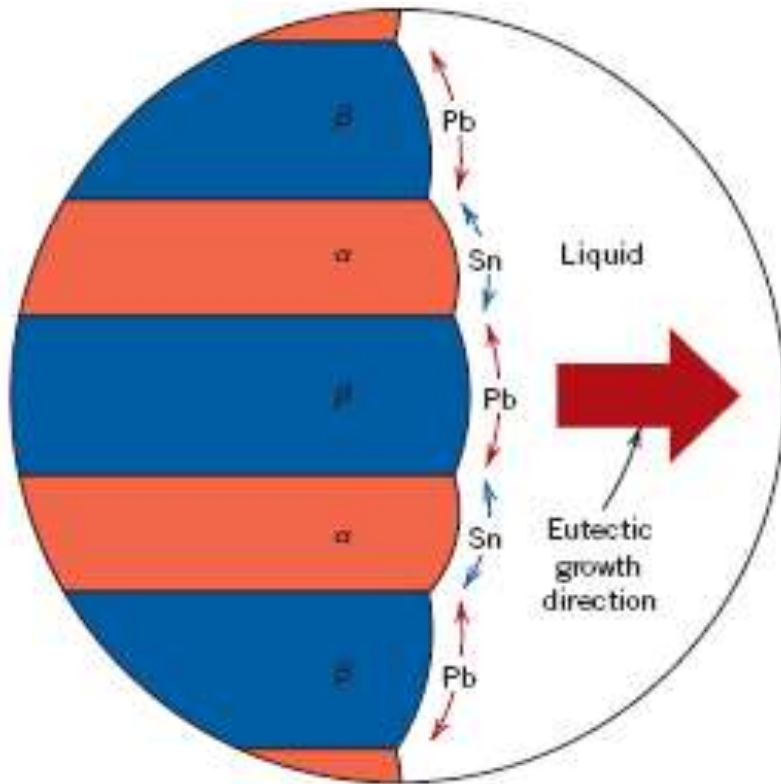
Micrograph of Pb-Sn  
eutectic  
microstructure



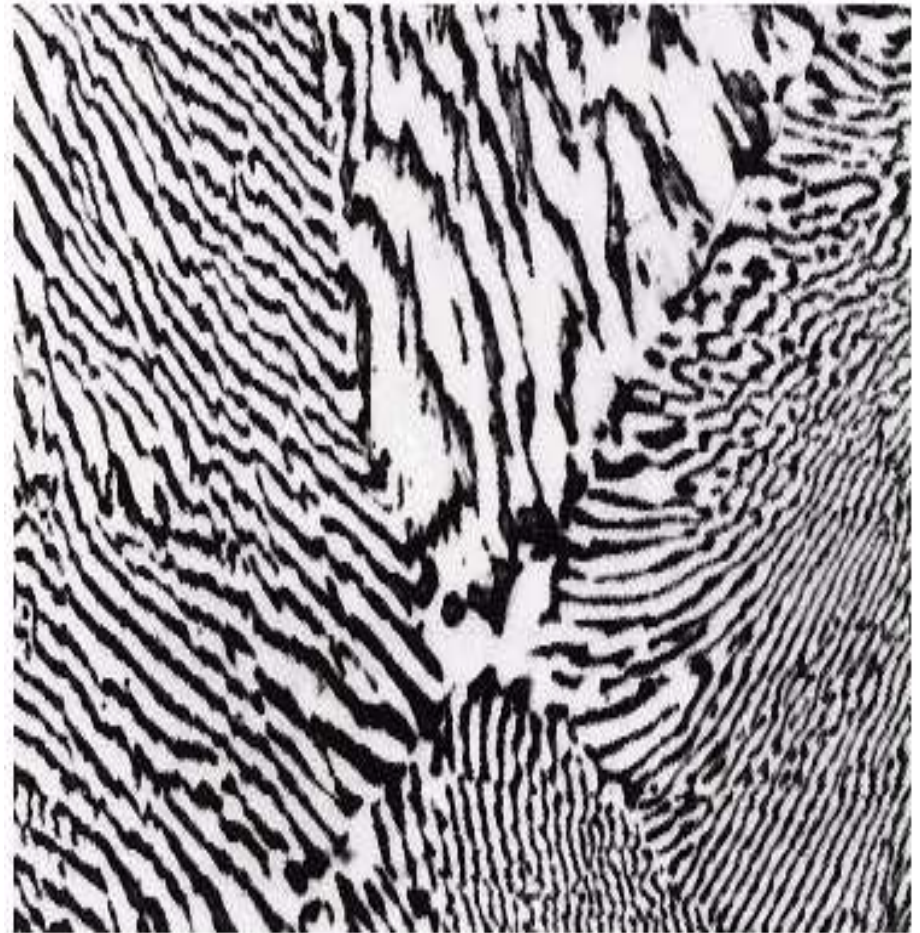
160  $\mu\text{m}$

Adapted from Fig. 9.14, Callister 7e.

# Lamellar Eutectic Structure

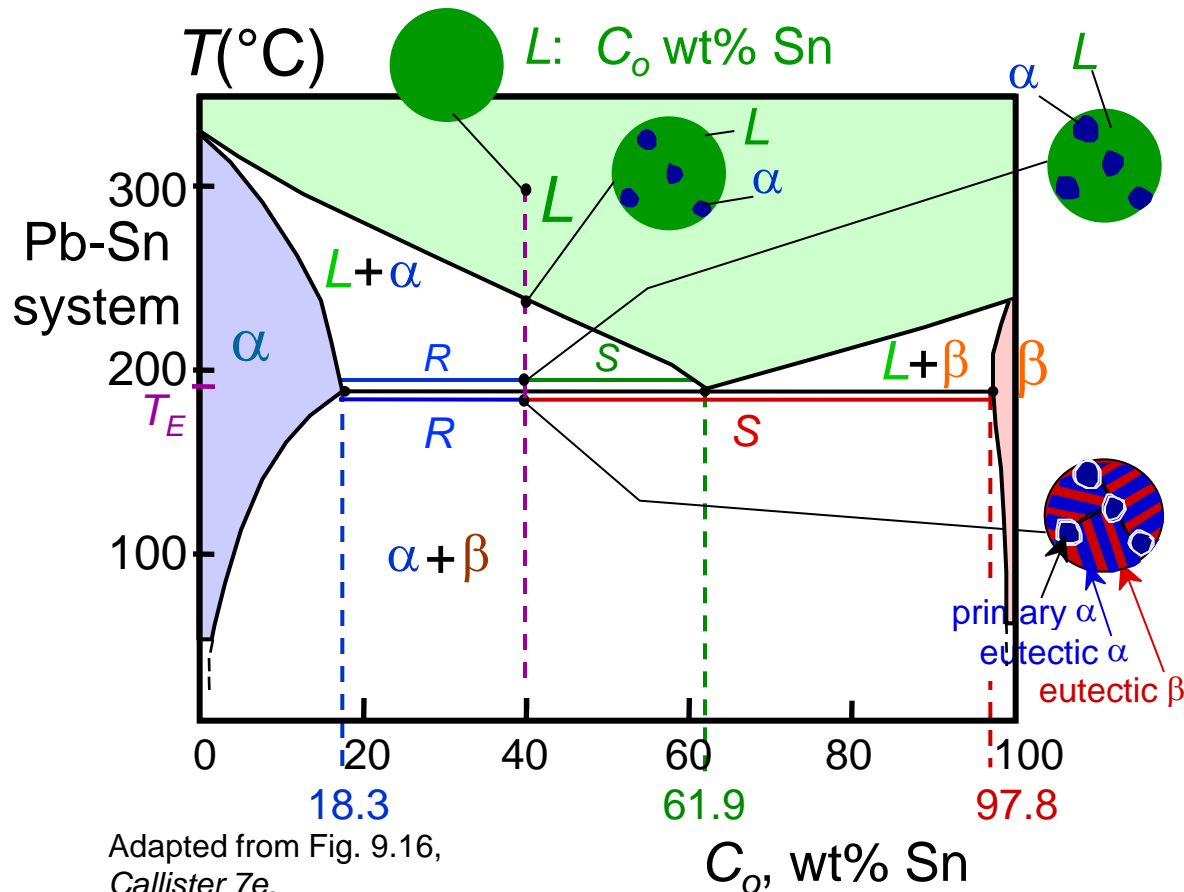


Adapted from Figs. 9.14 & 9.15, *Callister 7e*.



# Microstructures in Eutectic Systems: IV

- 18.3 wt% Sn <  $C_o$  < 61.9 wt% Sn
- Result:  $\alpha$  crystals and a eutectic microstructure



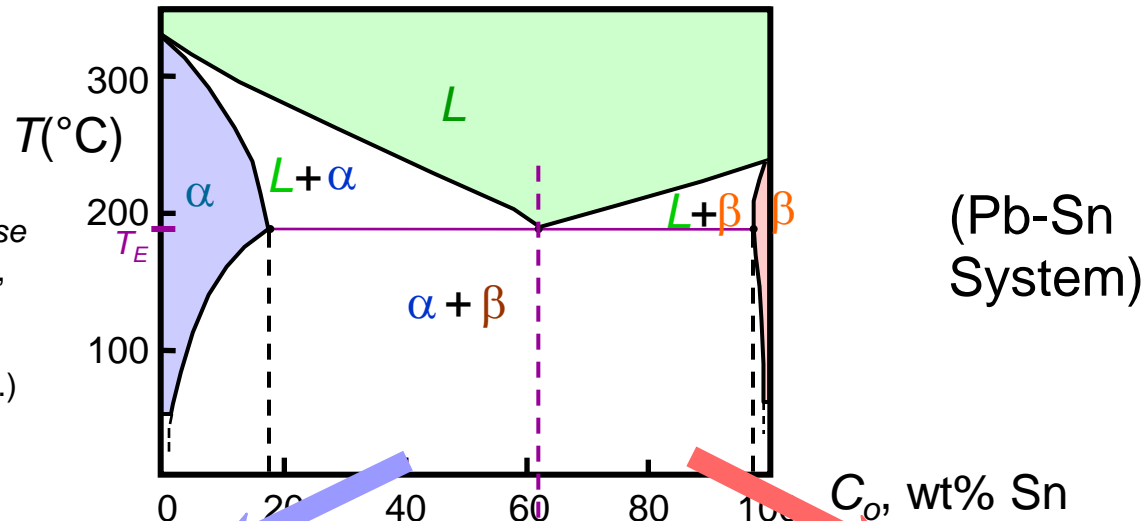
Adapted from Fig. 9.16,  
Callister 7e.

- Just above  $T_E$ :  
 $C_{\alpha} = 18.3$  wt% Sn  
 $C_L = 61.9$  wt% Sn  
 $W_{\alpha} = \frac{S}{R + S} = 50$  wt%  
 $W_L = (1 - W_{\alpha}) = 50$  wt%
- Just below  $T_E$ :  
 $C_{\alpha} = 18.3$  wt% Sn  
 $C_{\beta} = 97.8$  wt% Sn  
 $W_{\alpha} = \frac{S}{R + S} = 73$  wt%  
 $W_{\beta} = 27$  wt%

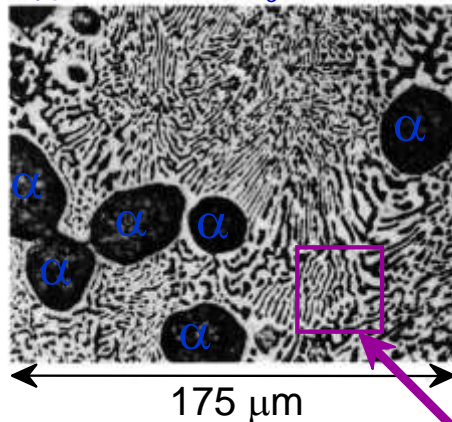


# Hypoeutectic & Hypereutectic

Adapted from Fig. 9.8, *Callister 7e*. (Fig. 9.8 adapted from *Binary Phase Diagrams*, 2nd ed., Vol. 3, T.B. Massalski (Editor-in-Chief), ASM International, Materials Park, OH, 1990.)



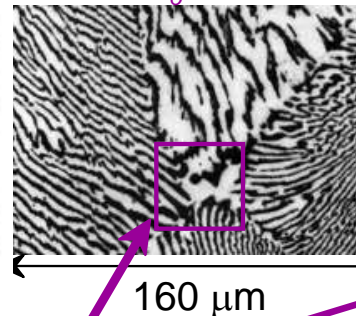
hypoeutectic:  $C_0 = 50 \text{ wt\% Sn}$



Adapted from Fig. 9.17, *Callister 7e*.

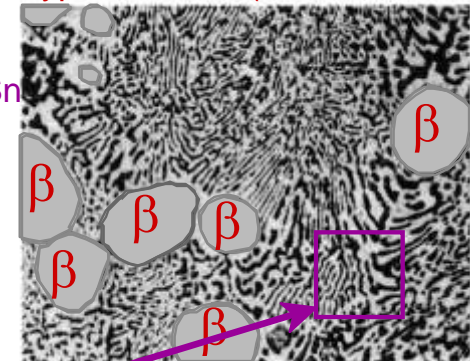
eutectic  
61.9

eutectic:  $C_0 = 61.9 \text{ wt\% Sn}$



eutectic micro-constituent  
Adapted from Fig. 9.14, *Callister 7e*.

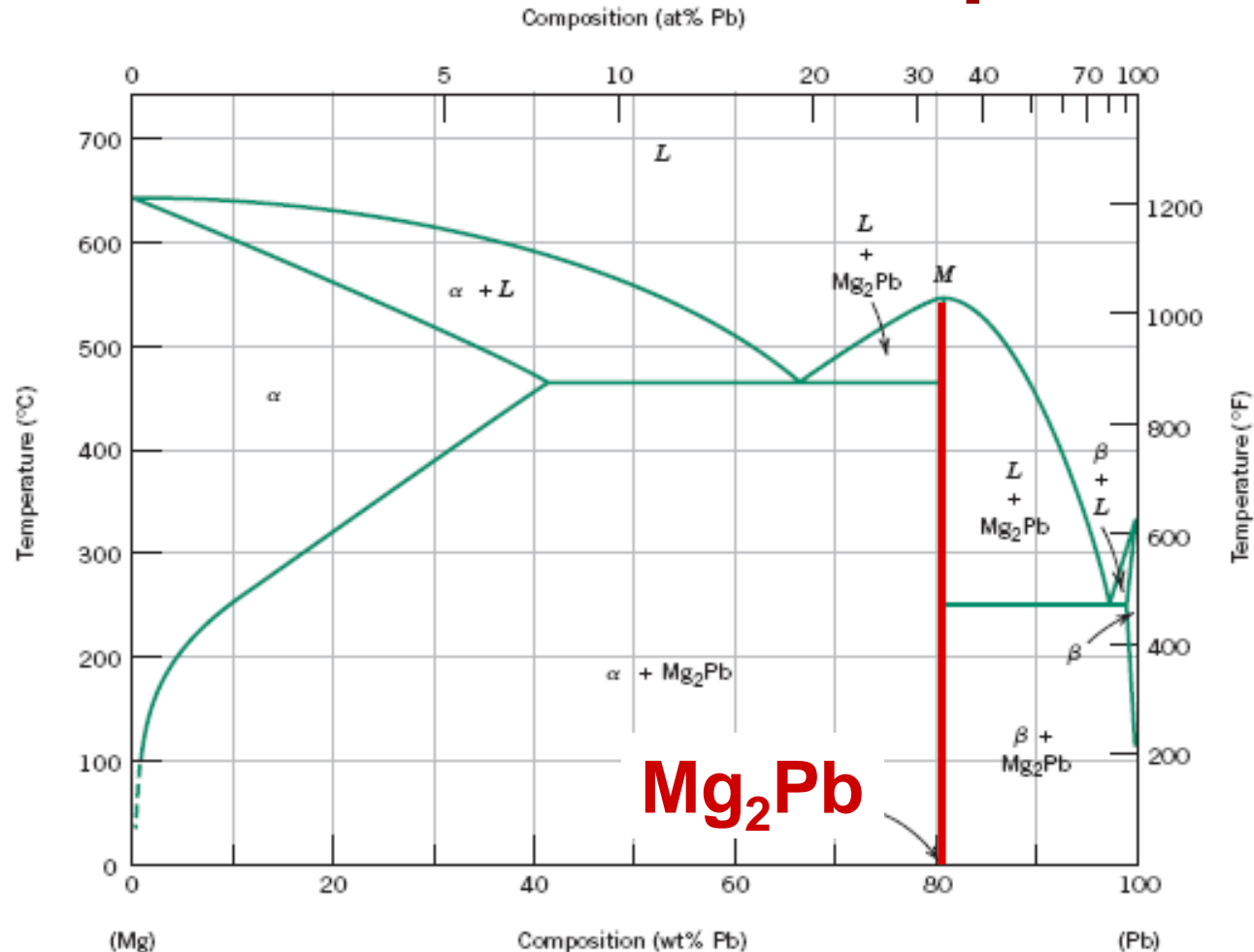
hypereutectic: (illustration only)



Adapted from Fig. 9.17, *Callister 7e*. (Illustration only)



# Intermetallic Compounds



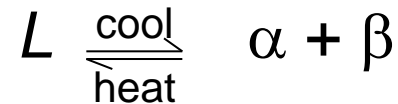
Adapted from  
Fig. 9.20, Callister 7e.

Note: intermetallic compound forms a line - not an area - because stoichiometry (i.e. composition) is exact.

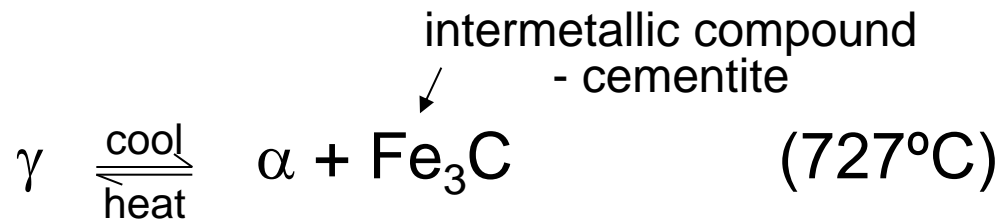
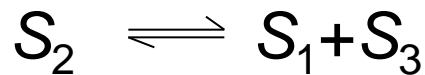


# Eutectoid & Peritectic

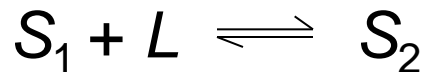
- **Eutectic** - liquid in equilibrium with two solids



- **Eutectoid** - solid phase in equilibrium with two solid phases



- **Peritectic** - liquid + solid 1  $\rightarrow$  solid 2 (Fig 9.21)

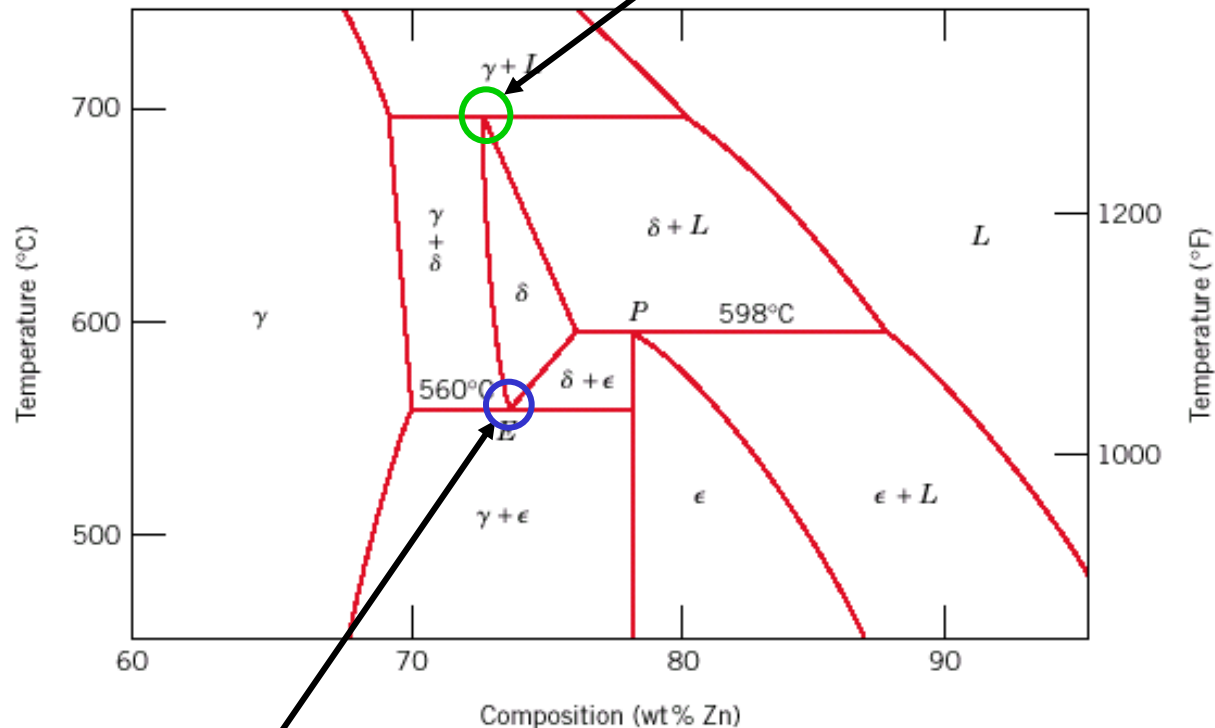




# Eutectoid & Peritectic

## Cu-Zn Phase diagram

Peritectic transition  $\gamma + L \rightleftharpoons \delta$



Eutectoid transition  $\delta \rightleftharpoons \gamma + \epsilon$

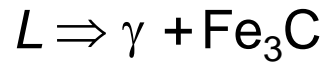
Adapted from  
Fig. 9.21, Callister 7e.



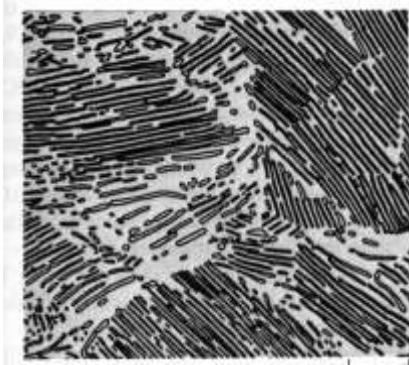
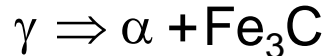
# Iron-Carbon (Fe-C) Phase Diagram

- 2 important points

-Eutectic (A):

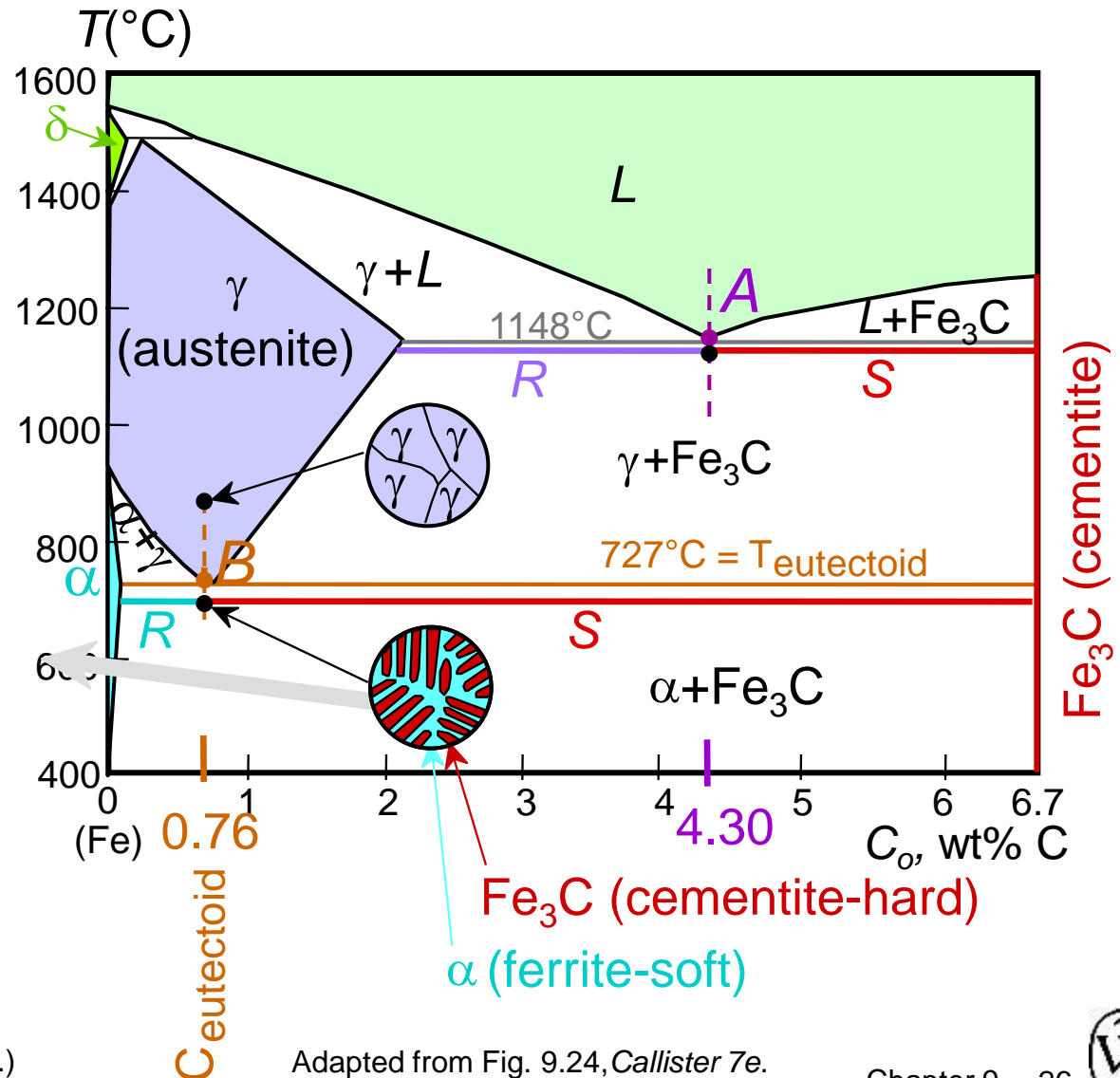


-Eutectoid (B):



120  $\mu\text{m}$

Result: Pearlite = alternating layers of  $\alpha$  and  $\text{Fe}_3\text{C}$  phases

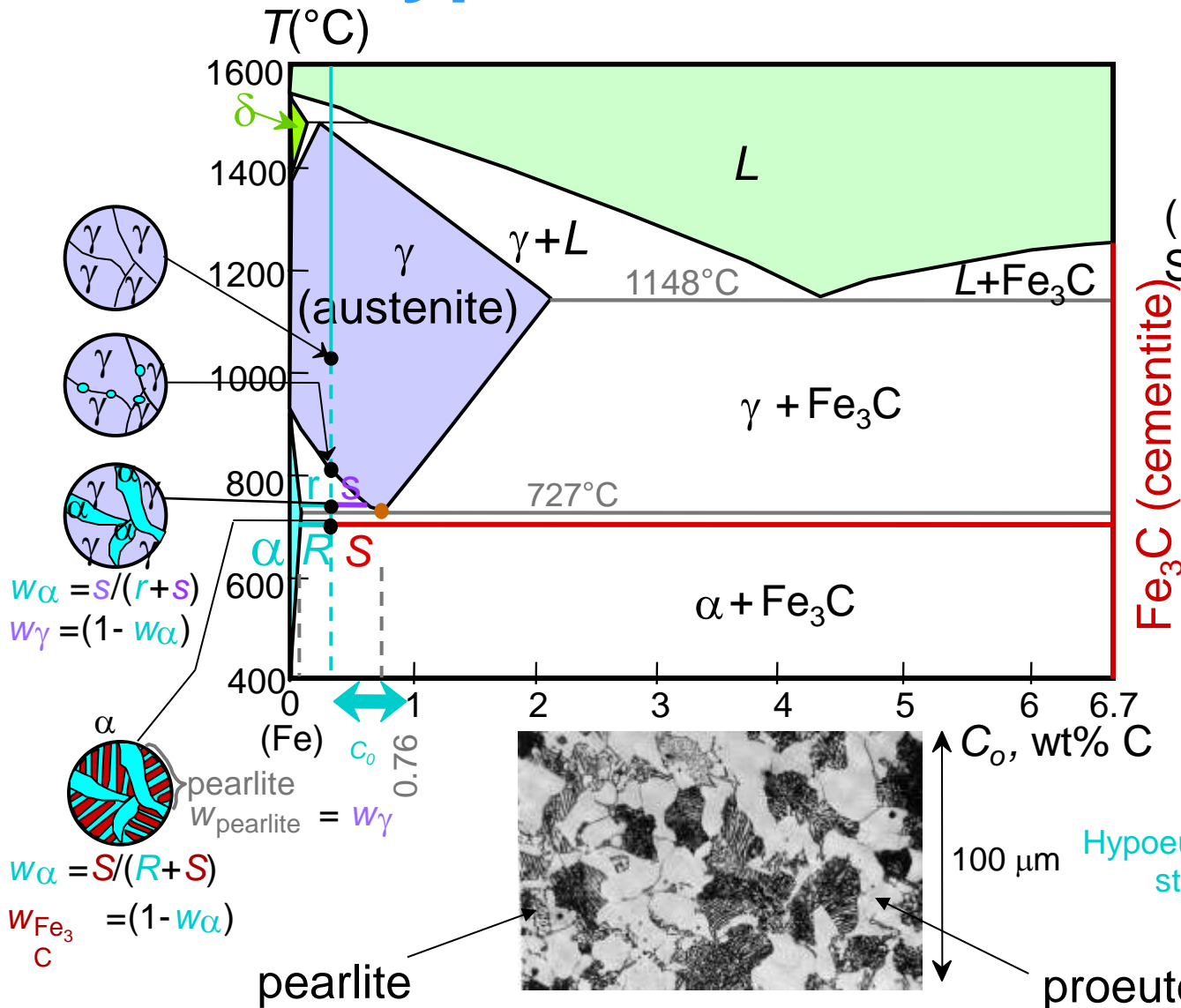


(Adapted from Fig. 9.27, Callister 7e.)

Adapted from Fig. 9.24, Callister 7e.



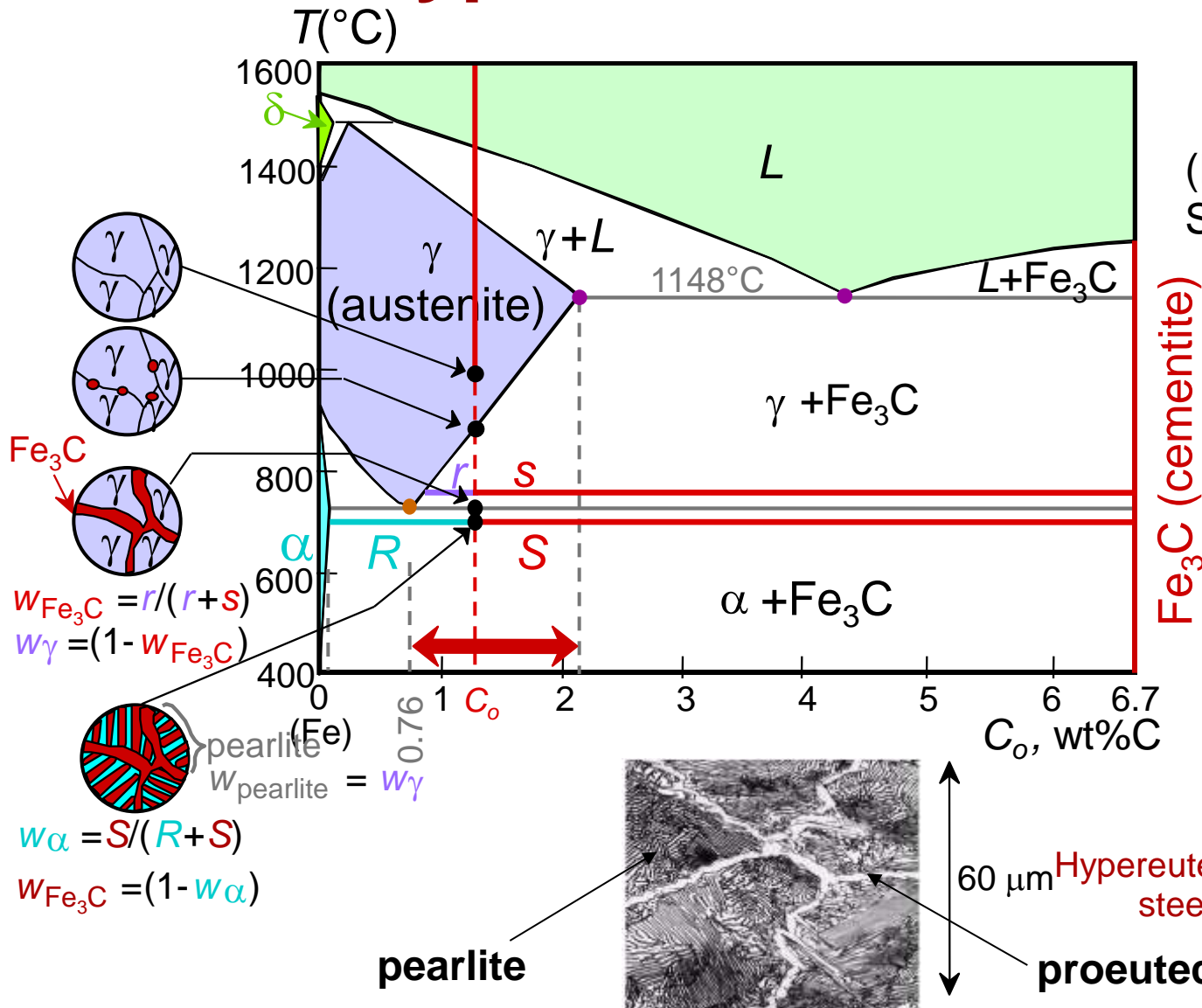
# Hypoeutectoid Steel



Adapted from Fig. 9.30, *Callister 7e*.



# Hypereutectoid Steel



(Fe-C System)

Adapted from Figs. 9.24 and 9.32, *Callister 7e*.  
(Fig. 9.24 adapted from *Binary Alloy Phase Diagrams*, 2nd ed., Vol. 1, T.B. Massalski (Ed.-in-Chief), ASM International, Materials Park, OH, 1990.)

Adapted from Fig. 9.33, *Callister 7e*.

Chapter 9 - 28



# Example: Phase Equilibria

For a 99.6 wt% Fe-0.40 wt% C at a temperature just below the eutectoid, determine the following

- a) composition of  $\text{Fe}_3\text{C}$  and ferrite ( $\alpha$ )
- b) the amount of carbide (cementite) in grams that forms per 100 g of steel
- c) the amount of pearlite and proeutectoid ferrite ( $\alpha$ )



# Chapter 9 – Phase Equilibria

**Solution:** a) composition of  $\text{Fe}_3\text{C}$  and ferrite ( $\alpha$ )

b) the amount of carbide  
(cementite) in grams that  
forms per 100 g of steel

$$C_o = 0.40 \text{ wt\% C}$$

$$C_\alpha = 0.022 \text{ wt\% C}$$

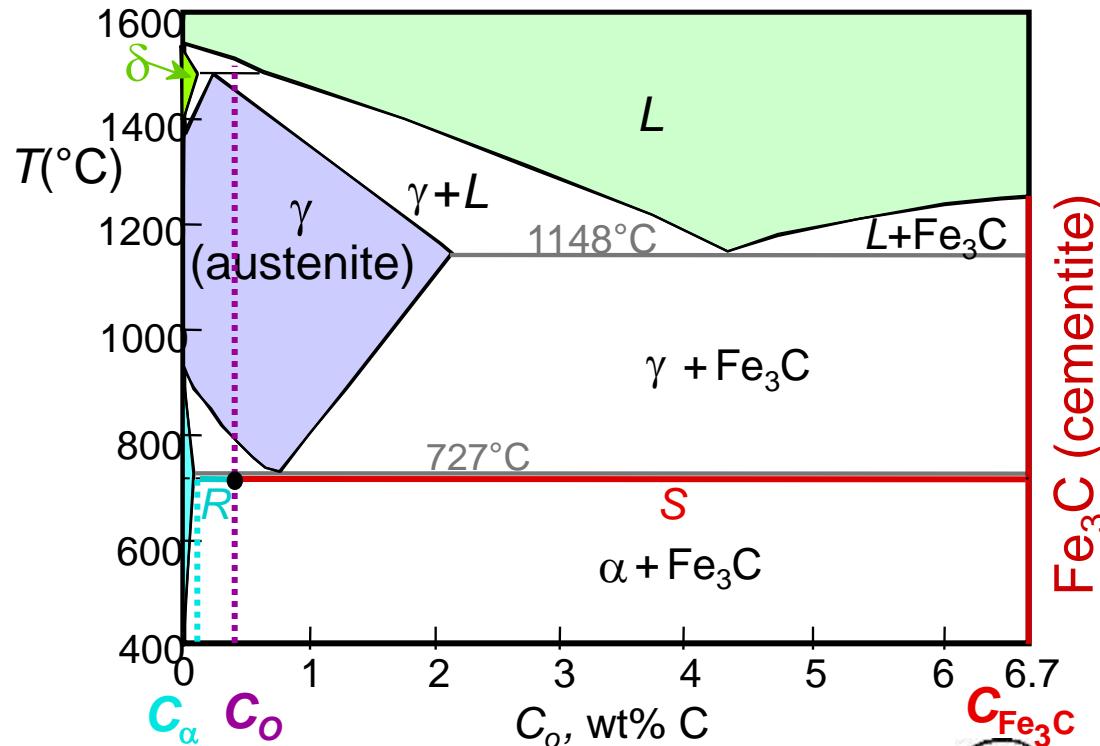
$$C_{\text{Fe}_3\text{C}} = 6.70 \text{ wt\% C}$$

$$\frac{\text{Fe}_3\text{C}}{\text{Fe}_3\text{C} + \alpha} = \frac{C_o - C_\alpha}{C_{\text{Fe}_3\text{C}} - C_\alpha} \times 100$$

$$= \frac{0.4 - 0.022}{6.7 - 0.022} \times 100 = 5.7\text{g}$$

$$\text{Fe}_3\text{C} = 5.7 \text{ g}$$

$$\alpha = 94.3 \text{ g}$$



# Chapter 9 – Phase Equilibria

- c. the amount of pearlite and proeutectoid ferrite ( $\alpha$ )

note: amount of pearlite = amount of  $\gamma$  just above  $T_E$

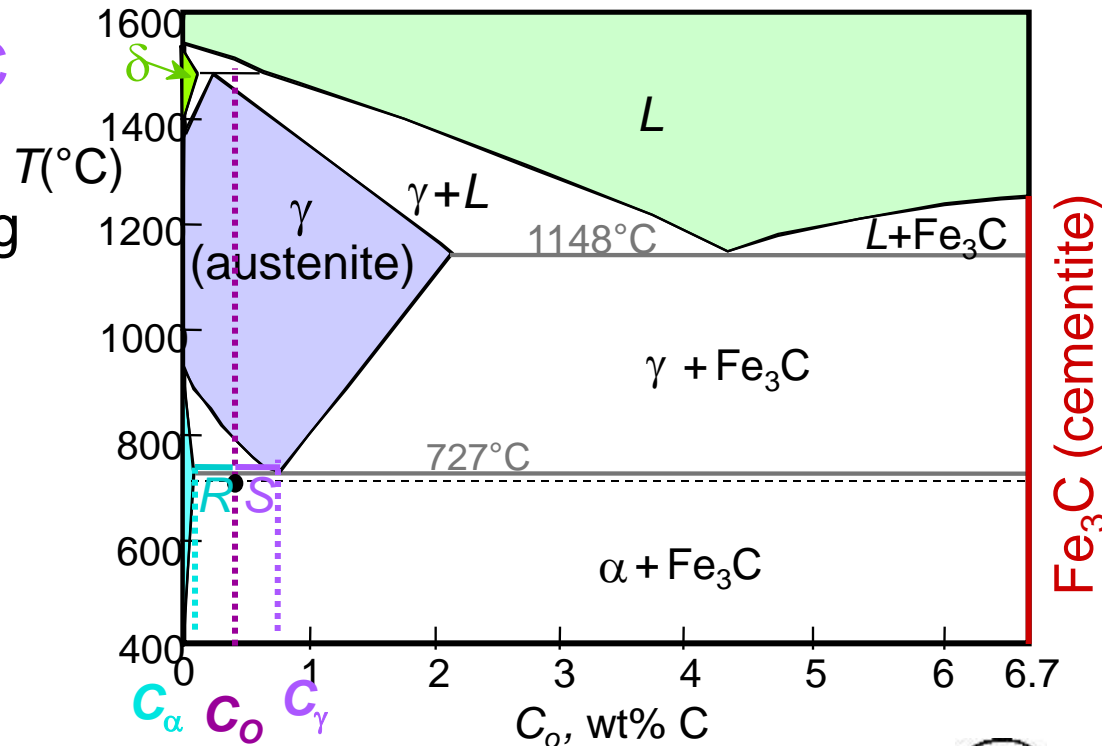
$C_o = 0.40 \text{ wt\% C}$

$$C_{\alpha} = 0.022 \text{ wt\% C}$$

$$C_{\text{pearlite}} = C_{\gamma} = 0.76 \text{ wt\% C}$$

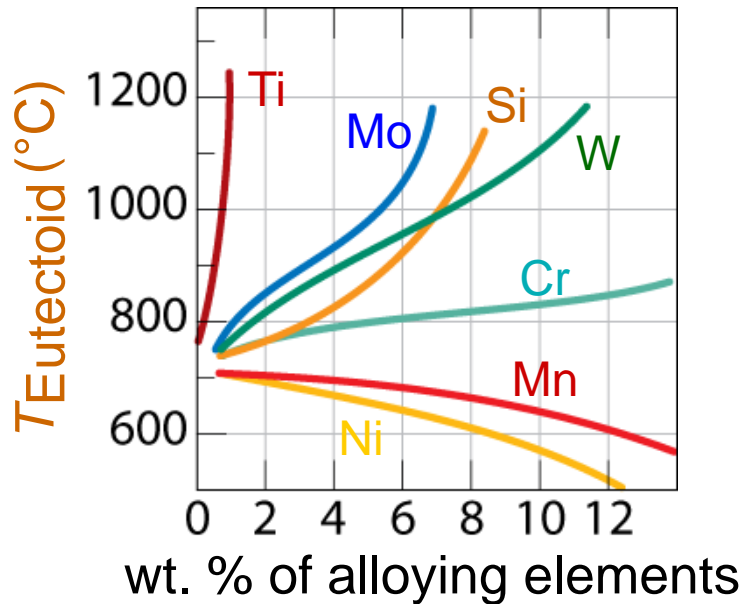
$$\frac{\gamma}{\gamma + \alpha} = \frac{C_o - C_\alpha}{C_\gamma - C_\alpha} \times 100 = 51.2 \text{ g}$$

pearlite = 51.2 g  
proeutectoid  $\alpha$  = 48.8 g



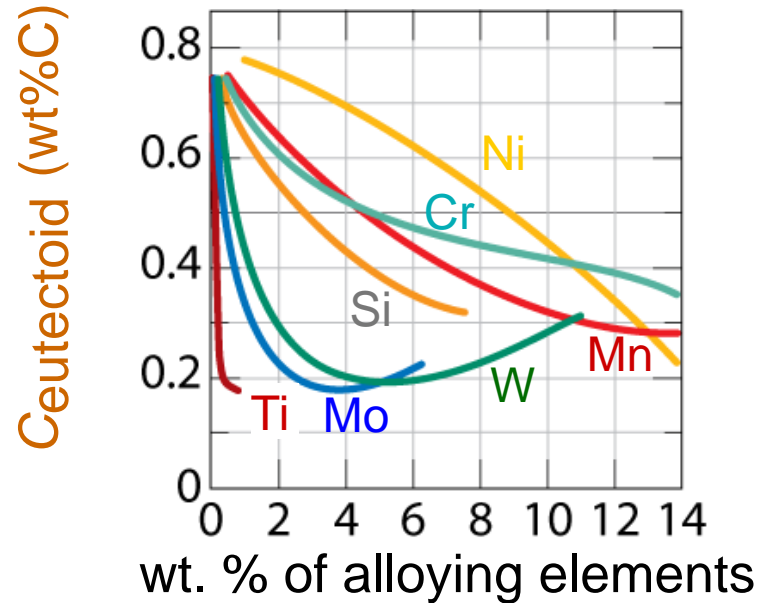
# Alloying Steel with More Elements

- $T_{\text{eutectoid}}$  changes:



Adapted from Fig. 9.34, Callister 7e. (Fig. 9.34 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)

- $C_{\text{eutectoid}}$  changes:



Adapted from Fig. 9.35, Callister 7e. (Fig. 9.35 from Edgar C. Bain, *Functions of the Alloying Elements in Steel*, American Society for Metals, 1939, p. 127.)





# Summary

- Phase diagrams are useful tools to determine:
  - the number and types of phases,
  - the wt% of each phase,
  - and the composition of each phasefor a given  $T$  and composition of the system.
- Alloying to produce a solid solution usually
  - increases the tensile strength ( $TS$ )
  - decreases the ductility.
- Binary eutectics and binary eutectoids allow for a range of microstructures.



# ANNOUNCEMENTS

Reading:

Core Problems:

Self-help Problems:

